

**Sheet #110. Circles and Ellipses: Implicit and Parametric**

To eliminate t, rewrite using  $\sin^2 t + \cos^2 t = 1$ .

Implicit equation is in terms of x and y only (part E).

\*USE Calculator. Zoom 6 then Zoom 5.

A-B. Graph and write implicit formula (eliminate t).

Find center (h, k) and radius, r.

A.  $x = 4 + \cos t, y = -5 + \sin t$ .

$$(x-4)^2 = \cos^2 t$$

$$(y+5)^2 = \sin^2 t$$

$$(x-4)^2 + (y+5)^2 = 1$$

$$\boxed{r=1}$$

$$\boxed{(h,k) = (4, -5)}$$

B.  $x = 1 - 2\cos t, y = 6 + 2\sin t$ .

$$(x-1)^2 = 4\cos^2 t$$

$$(y-6)^2 = 4\sin^2 t$$

$$\boxed{(x-1)^2 + (y-6)^2 = 4}$$

$$\boxed{r=2}$$

$$\boxed{(h,k) = (1, 6)}$$

C-D. Graph and write implicit formula. Find and mark these points on the graph: center, vertices, and co-vertices.

C.  $x = 9\cos t, y = 5\sin t$ .

$$\left(\frac{x}{9}\right)^2 = \cos^2 t$$

$$\left(\frac{y}{5}\right)^2 = \sin^2 t$$

$$\boxed{\left(\frac{x}{9}\right)^2 + \left(\frac{y}{5}\right)^2 = 1}$$

$$\boxed{(h,k) = (0,0)}$$

$$a=9 \text{ Vert.} = (\pm 9, 0)$$

$$b=5 \text{ Covert.} = (0, \pm 5)$$

$$\text{Foci} = (\pm 2\sqrt{14}, 0)$$

D.  $x = 3 - 2\sin t, y = -1 + 3\cos t$ . Be careful with vertices.

$$(x-3)^2 = 4\sin^2 t$$

$$(y+1)^2 = 9\cos^2 t$$

$$\boxed{\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1}$$

$$\boxed{(h,k) = (3, -1)}$$

$$a=3 \text{ Vertices: } (3, 2)$$

$$\uparrow \text{Vertical is larger. } (3, -4)$$

$$b=2 \text{ Covert. } (5, -1)$$

$$\uparrow \text{Horizontal } (1, -1)$$

$$\text{or } (3, -1 \pm 3) \text{ and } (3 \pm 2, -1)$$

E. Write this implicit equation in parametric form using t. Graph it. Find and mark these points on the graph: center, vertices, and co-vertices.

$$\frac{(x-2)^2}{16} + \frac{(y-3)^2}{25} = 1$$

$$\frac{x-2}{4} = \cos t$$

$$\frac{y-3}{5} = \sin t$$

$$a=5 \text{ Vert.}$$

$$b=4$$

$$\boxed{x = 2 + 4\cos t}$$

$$\boxed{y = 3 + 5\sin t}$$

