

12-3 Trigonometric Functions of General Angles

Objective: To define trigonometric functions of general angles.

Vocabulary

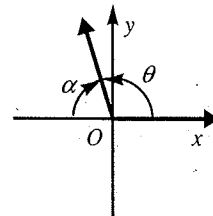
Reference angle If θ is not a quadrantal angle, there is a unique acute angle α , corresponding to θ , such that either $\theta + \alpha$ or $\theta - \alpha$ is an integral multiple of 180° . α is called the reference angle of θ .

Example: If θ measures 115° , its reference angle, α , measures 65° .

Formulas For any angle θ and any point $P(x, y)$ on the terminal side of θ :

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x}, \text{ if } x \neq 0$$

$$\csc \theta = \frac{r}{y}, \text{ if } y \neq 0 \qquad \sec \theta = \frac{r}{x}, \text{ if } x \neq 0 \qquad \cot \theta = \frac{x}{y}, \text{ if } y \neq 0$$



Example 1 Find the values of the six trigonometric functions of an angle θ in standard position whose terminal side passes through $(-5, -12)$.

Solution

First make a sketch.

Here $x = -5$ and $y = -12$.

Since r is the distance between $(0, 0)$ and $(-5, -12)$, you can find r using the distance formula.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(-5 - 0)^2 + (-12 - 0)^2}$$

$$r = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$\sin \theta = -\frac{12}{13}$$

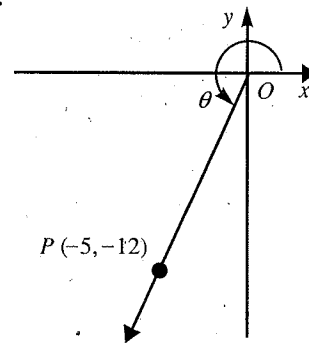
$$\cos \theta = -\frac{5}{13}$$

$$\tan \theta = \frac{-12}{-5} = \frac{12}{5}$$

$$\csc \theta = -\frac{13}{12}$$

$$\sec \theta = -\frac{13}{5}$$

$$\cot \theta = \frac{-5}{-12} = \frac{5}{12}$$



Find the values of the six trigonometric functions of an angle θ in standard position whose terminal side passes through point P . If any value is undefined, say so.

1. $P(4, -3)$ 2. $P(-12, 5)$ 3. $P(-2, -2)$ 4. $P(15, -8)$ 5. $P(0, 1)$

Example 2

Find the measure of the reference angle α for each given angle θ .

a. $\theta = 155^\circ$

b. $\theta = 310^\circ 30'$

c. $\theta = -125^\circ$

Solution

a. Since θ is in quadrant II: $\alpha = 180^\circ - \theta = 180^\circ - 155^\circ = 25^\circ$

b. Since θ is in quadrant IV:

$$\alpha = 360^\circ - \theta = 360^\circ - 310^\circ 30' = 359^\circ 60' - 310^\circ 30' = 49^\circ 30'$$

c. First find the positive angle that is coterminal with -125° :

$$-125^\circ + 360^\circ = 235^\circ$$

Since 235° is in quadrant III: $\alpha = \theta - 180^\circ = 235^\circ - 180^\circ = 55^\circ$

12-3 Trigonometric Functions of General Angles (continued)

Find the measure of the reference angle α of the given angle θ .

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|-----------------------------|------------------------------|------------------------------|------------------------------|
| 6. $\theta = 250^\circ$ | 7. $\theta = 172^\circ$ | 8. $\theta = -200^\circ$ | 9. $\theta = -142^\circ$ |
| 10. $\theta = 730^\circ$ | 11. $\theta = 480^\circ$ | 12. $\theta = 98.2^\circ$ | 13. $\theta = -166.2^\circ$ |
| 14. $\theta = -288.6^\circ$ | 15. $\theta = 149^\circ 20'$ | 16. $\theta = 243^\circ 15'$ | 17. $\theta = 300^\circ 50'$ |

Example 3 Write $\sin 230^\circ$ as a function of an acute angle.

Solution The reference angle of 230° is $230^\circ - 180^\circ = 50^\circ$.
 Since 230° is in the third quadrant, its sine is negative (see note below).
 $\therefore \sin 230^\circ = -\sin 50^\circ$.

Note: The signs of the trigonometric functions vary with the quadrants. All values are positive in the first quadrant. Sine and cosecant are positive in the second quadrant. Tangent and cotangent are positive in the third quadrant. Cosine and secant are positive in the fourth quadrant.

Write each of the following as a function of an acute angle.

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|------------------------|---------------------------|--------------------------|--------------------------|
| 18. $\sin 312^\circ$ | 19. $\cos 215^\circ$ | 20. $\tan (-163^\circ)$ | 21. $\sin (-29^\circ)$ |
| 22. $\sec 276.8^\circ$ | 23. $\csc (-198.3^\circ)$ | 24. $\cot 312^\circ 15'$ | 25. $\cos 259^\circ 12'$ |

Example 4 Find the exact values of the six trigonometric functions of 225° . Use the table of exact trigonometric values found in Lesson 12-2 of your textbook.

Solution The reference angle of 225° is 45° .
 Since 225° is in the third quadrant, only the tangent and cotangent are positive.

$$\sin 225^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2} \quad \csc 225^\circ = -\csc 45^\circ = -\sqrt{2}$$

$$\cos 225^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2} \quad \sec 225^\circ = -\sec 45^\circ = -\sqrt{2}$$

$$\tan 225^\circ = \tan 45^\circ = 1 \quad \cot 225^\circ = \cot 45^\circ = 1$$

Find the exact values of the six trigonometric functions of each angle.

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|-----------------|------------------|-----------------|------------------|-----------------|
| 26. 120° | 27. -135° | 28. 210° | 29. -225° | 30. 420° |
|-----------------|------------------|-----------------|------------------|-----------------|

Mixed Review Exercises

Find the zeros of each function. If the function has no zeros, say so.

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|--------------------------------------|--------------------------|--------------------------|
| 1. $f(x) = 2x + 6$ | 2. $g(x) = \log(x - 2)$ | 3. $h(x) = 6x^2 + x - 2$ |
| 4. $F(x) = \frac{x^3 - 9x}{x^2 + 2}$ | 5. $G(x) = x^2 + 3x + 8$ | 6. $H(x) = 2^x$ |