NAME:

# Triangle Trigonometry

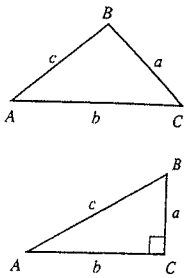
## 12-5 Solving Right Triangles

**Objective** To find the sides and angles of a right triangle.

Any triangle $ABC$ has six measurements associated with it: the lengths of the three sides, denoted by $a$, $b$, and $c$, and the measures of the three angles, denoted by $\angle A$, $\angle B$, and $\angle C$. (Note that the lower-case letters denote the lengths of sides opposite angles labeled by the corresponding capital letters.) If any three of these measurements (other than the three angle measures) are given, then the other three can be found.

Finding measurements for all the sides and angles of a triangle is called **solving the triangle.**

In the case of a right triangle $ABC$, one angle is 90°. You can solve the triangle if the lengths of two sides, or the length of one side and the measurement of one acute angle, are known.




**Example 1** Solve the right triangle shown above if $\angle A = 36°$ and $b = 50$.

**Solution 1** Since $\angle A + \angle B = 90°$, $\angle B = 90° - \angle A = 90° - 36° = 54°$.

$$\cot A = \frac{b}{a} \qquad \cos A = \frac{b}{c}$$

$$\cot 36° = \frac{50}{a} \qquad \cos 36° = \frac{50}{c}$$

$$a = \frac{50}{\cot 36°} \qquad c = \frac{50}{\cos 36°}$$

$$= \frac{50}{1.376} \qquad = \frac{50}{0.8090}$$

$$= 36.3 \qquad = 61.8$$

$\therefore \angle B = 54°$, $a = 36.3$, and $c = 61.8$ *Answer*

**Solution 2** Again $\angle B = 90° - 36° = 54°$.

$$\tan A = \frac{a}{b} \qquad \sec A = \frac{c}{b}$$

$$\tan 36° = \frac{a}{50} \qquad \sec 36° = \frac{c}{50}$$

$$a = 50(\tan 36°) \qquad b = 50(\sec 36°)$$

$$= 50(0.7265) \qquad = 50(1.236)$$

$$= 36.3 \qquad = 61.8$$

$\therefore \angle B = 54°$, $a = 36.3$, and $c = 61.8$ *Answer*

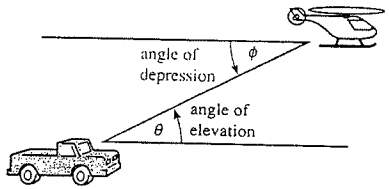


In Example 1, Solution 1 or Solution 2 are both well suited for calculator use. However, if tables are used, Solution 2 would be a better choice because long division is avoided.

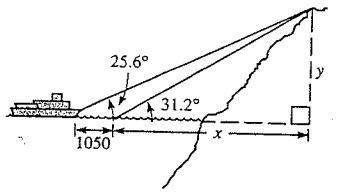
Measurements of lengths and angles are approximations, as are the values given by tables and calculators. The following is a guide to the corresponding accuracies of length and angle measurements.

| An angle measured to | | A length measured to |
|---|---|---|
| 1° | | 2 significant digits. |
| 0.1° or 10′ | corresponds to | 3 significant digits. |
| 0.01° or 1′ | | 4 significant digits. |

The figure at the right shows a helicopter and a pickup truck. To see the helicopter, the driver's line of sight must be raised, or *elevated*, at an angle $\theta$ above the horizontal. This angle $\theta$ is called the **angle of elevation** of the helicopter. Similarly, the angle $\phi$ is called the **angle of depression** of the truck from the helicopter. The angle of elevation and the corresponding angle of depression always have the same measure.




**Example 2** A research ship finds that the angle of elevation of a volcanic island peak is 25.6°. After the ship has moved 1050 m closer to the island, the angle of elevation is 31.2°. What is the height of the peak above sea level? (Note that the sketch shown at the right is not drawn to scale.)




**Solution**

$$\cot 31.2° = \frac{x}{y} \quad \text{and} \quad \cot 25.6° = \frac{1050 + x}{y}$$

$$x = y \cot 31.2° \qquad y \cot 25.6° = 1050 + x$$

$$x = 1.651y \qquad 2.087y = 1050 + x$$

Substitute $1.651y$ for $x$ in the second equation.

$$2.087y = 1050 + 1.651y$$

$$0.436y = 1050$$

$$y = \frac{1050}{0.436} = 2410 \text{ (to three significant digits)}$$

$\therefore$ the volcanic peak is 2410 m above sea level. *Answer*



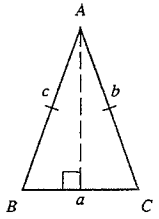
Structure & Method, Book 2.

5

The altitude drawn in the diagram at the right below divides isosceles triangle $ABC$ into two congruent right triangles. Constructing such an altitude will help you to solve any isosceles triangle.

**Example 3** Solve isosceles triangle $ABC$ if $a = 31.0$ and $c = 42.5$.

**Solution** Make a sketch as shown.




In the right triangle on the left in the diagram, the side adjacent to $\angle B$ has length $\frac{1}{2}a$, or 15.5. So

$$\cos B = \frac{15.5}{42.5} = 0.3647 \text{ and } \angle B = 68.6°.$$

Since $\Delta ABC$ is isosceles,

$\angle C = \angle B = 68.6°$ and $b = c = 42.5$

$\angle A = 180° - (\angle B + \angle C)$

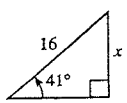
$= 180° - 137.2° = 42.8°$

$\therefore \angle B = 68.6°, \angle C = 68.6°, \angle A = 42.8°$, and $b = 42.5$ ***Answer***

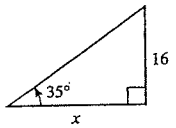
## Oral Exercises

**Give an equation that can be used to find the value of $x$.**

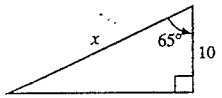
**1.**



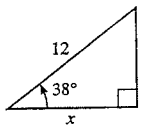

**2.**



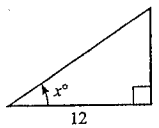

**3.**



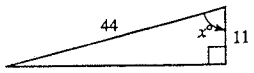

**4.**




**5.**




**6.**




**7.** The angle of depression of $Q$ from $P$ measures 25°. Therefore, the angle of elevation of $P$ from $Q$ measures ___?___.

**8.** What fact from geometry did you use in answering the question in Exercise 7?

**9.** Give each angle to the nearest 10′.

**a.** 42°13′21″ **b.** 140°9′ **c.** 0°11′59″

**10.** Give each number to three significant digits.

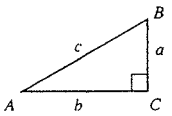
**a.** 6.758 **b.** 1284.56 **c.** 0.39975



## Written Exercises

**Give lengths to three significant digits and angle measures to the nearest tenth of a degree or nearest ten minutes. You may wish to use a calculator.**

**Solve each right triangle $ABC$.**




**A**

**1.** $\angle A = 36.2°, c = 68$

**2.** $\angle B = 15.8°, c = 12.2$

**3.** $\angle B = 65.4°, a = 2.35$

**4.** $\angle A = 82.1°, b = 246$

**5.** $\angle B = 48.3°, b = 74.7$

**6.** $\angle A = 24.0°, a = 5.25$

**7.** $a = 230, c = 320$

**8.** $a = 52.5, b = 28.0$

**9.** $a = 0.123, b = 0.315$

**10.** $b = 3.90, c = 42.5$

**11.** $\angle B = 58°10', c = 420$

**12.** $\angle A = 38°40', c = 42.5$

**13.** $\angle A = 15°30', a = 4.50$

**14.** $\angle B = 67°20', a = 450$

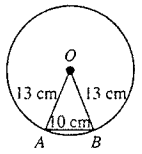
**15.** $\angle A = 30°50', b = 53.5$

**16.** $\angle B = 85°10', b = 0.620$

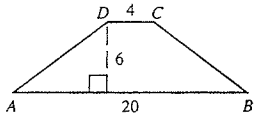
$1° = 60' = 3600''$

**B**

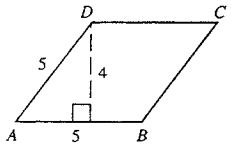
**17.** The radius of circle $O$ is 13 cm and the length of $\overline{AB}$ is 10 cm. Find the measure of $\angle AOB$.




Ex. 17




Ex. 18




Ex. 19

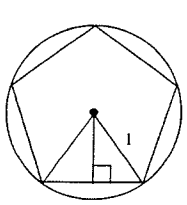
**18.** The height of an isosceles trapezoid is 6 units and the bases have lengths 4 units and 20 units. Find the measures of the angles.

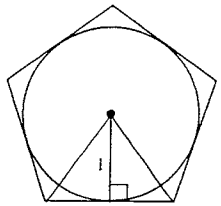
**19.** A rhombus has sides 5 units long and its height is 4 units. Find its angles.

**20.** **a.** Find the perimeter of a regular pentagon inscribed in a unit circle.

**b.** Find the perimeter of a regular pentagon circumscribed about a unit circle.

(*Hint:* Use the shaded right triangles in the figures below.)



21. Repeat Exercise 20 for a regular polygon having 10 sides.
22. Repeat Exercise 20 for a regular polygon having 20 sides.
23. Explain why the number $2\pi$ lies between the answers to the (a) and (b) parts of Exercises 20, 21, and 22.

**C** 24. **a.** Repeat Exercise 20 for a polygon having $n$ sides.
**b.** Use the results of part (a) to tell what number is approached by $n \sin\left(\frac{180}{n}\right)^\circ$ as $n$ gets larger and larger. What number is approached by $n \tan\left(\frac{180}{n}\right)^\circ$ as $n$ gets larger and larger?

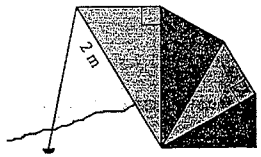
## Problems

**Give lengths to three significant digits and angle measures to the nearest tenth of a degree. You may wish to use a calculator.**

**A** 1. What is the angle of elevation of the sun when a tree 6.25 m tall casts a shadow 10.1 m long?
2. A boy flying a kite is standing 30 ft from a point directly under the kite. If the string to the kite is 50 ft long, find the angle of elevation of the kite.
3. A cable 4 m long is attached to a pole. The cable is staked to the ground 1.75 m from the base of the pole. Find the angle that the cable makes with the ground.
4. How far from the base of a building is the bottom of a 30 ft ladder that makes an angle of 75° with the ground?
5. The angle of elevation of the summit of a mountain from the bottom of a ski lift is 33°. A skier rides 1000 ft on this ski lift to get to the summit. Find the vertical distance between the bottom of the ski lift and the summit.
6. The approach pattern to an airport requires pilots to set an 11° angle of descent toward the runway. If a plane is flying at an altitude of 9500 m, at what distance (measured along the ground) from the airport must the pilot start the descent?
7. The distance from the point directly under a hot air balloon to the point where the balloon is staked to the ground with a rope is 285 ft. The angle of elevation up the rope to the balloon is 48°. Find the height of the balloon.
8. Opposite corners of a small rectangular park are joined by diagonal paths, each 360 m long. What are the dimensions of the park if the paths intersect at a 65° angle?

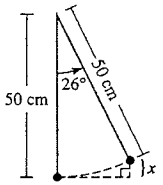


**B** 9. A pendulum in a grandfather clock is 160 cm long. The horizontal distance between the farthest points in a complete swing is 65 cm. Through what angle does the pendulum swing?
10. A camping tent is supported by a rope stretched between two trees at a height of 210 cm. If the sides of the tent make an angle of 55° with the level ground, how wide is the tent at the bottom?
11. From the top of a 135 ft observation tower, a park ranger sights two forest fires on opposite sides of the tower. If their angles of depression are 42.5° and 32.6°, how far apart are the fires?
12. From a point 250 m from the base of a vertical cliff, the angles of elevation to the top and bottom of a radio tower on top of the cliff are 62.2° and 59.5°. How tall is the tower?



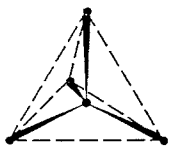
Ex. 13

13. The side of the beach shelter shown at the right is made up of four 30°–60°–90° triangles. Find the dimension marked $x$.
14. A pendulum 50 cm long is moved 26° from the vertical. How much is the lower end of the pendulum raised?



Ex. 14

**C** 15. Two observers 1600 m apart on a straight, flat road measure the angles of elevation of a helicopter hovering over the road between them. If these angles are 32.0° and 50.5°, how high is the helicopter?
16. The pilot of a hot air balloon sees a field straight ahead that he knows is 1000 ft long. The angles of depression to the ends of the field are 25.4° and 34.7°. What is the height of the balloon and its horizontal distance from the nearer end of the field?
17. In a molecule of carbon tetrachloride, the four chlorine atoms are at the vertices of a regular tetrahedron, with the carbon atom in the center. What is the angle between two of the carbon-chlorine bonds (shown in red in the figure)?


Ex. 17

## Mixed Review Exercises

**Give the exact values of the six trigonometric functions of each angle. If any function is not defined for the angle, say so.**

1. 450°
2. −135°
3. 330°
4. −240°

**Write in simplest form without negative exponents.**

5. $\frac{x^{-1}-1}{x-x^{-1}}$
6. $\sqrt{50m^4}$
7. $\frac{1}{y+2}+\frac{4}{y^2-4}$
8. $(-5a^3b)^2(2a^2b)^3$
9. $(x^{2/3})^{-3/4}$
10. $(2u-3)(u^2+u-2)$