

Name = \_\_\_\_\_

# Precalculus

## Trigonometric Functions

I. AMC

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- I. AMC (Merrill) Practice Worksheets
- II. FMC (Connally) Exercises & Problems
- III. PTCA (Foerster) Explorations
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# Practice Worksheet

## Angles and Their Measure

If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

1.  $\frac{7\pi}{12}$

2.  $-\frac{2\pi}{3}$

3.  $371^\circ$

4.  $\frac{14\pi}{5}$

5.  $-156^\circ$

6.  $1000^\circ$

7.  $332^\circ$

8.  $-240^\circ$

Change each degree measure to radian measure in terms of  $\pi$ .

9.  $36^\circ$

10.  $-250^\circ$

11.  $-145^\circ$

12.  $6^\circ$

13.  $870^\circ$

14.  $18^\circ$

15.  $-820^\circ$

16.  $345^\circ$

Change each radian measure to degree measure.

17.  $-1$

18.  $4\pi$

19.  $-2.56$

20.  $12.85$

21.  $\frac{3\pi}{16}$

22.  $-\frac{7\pi}{9}$

23.  $\frac{13\pi}{30}$

24.  $-\frac{17\pi}{3}$

Find one positive angle and one negative angle that are coterminal with each angle.

25.  $70^\circ$

26.  $-\frac{2\pi}{5}$

27.  $-300^\circ$

28.  $\frac{3\pi}{4}$

Find the reference angle for each angle with the given measure.

29.  $-20^\circ$

30.  $160^\circ$

31.  $-545^\circ$

32.  $300^\circ$

33.  $\frac{10\pi}{3}$

34.  $-\frac{5\pi}{8}$

35.  $-\frac{\pi}{4}$

36.  $-\frac{7\pi}{3}$

# Practice Worksheet

## Central Angles and Arcs

Given the radian measure of a central angle, find the measure of its intercepted arc in terms of  $\pi$  in a circle of radius 10 cm.

1.  $\frac{\pi}{6}$

2.  $\frac{\pi}{3}$

3.  $\frac{\pi}{2}$

4.  $\frac{\pi}{5}$

5.  $\frac{3\pi}{5}$

6.  $\frac{4\pi}{7}$

7.  $\frac{\pi}{12}$

8.  $\frac{\pi}{24}$

Given the measurement of a central angle, find the measure of its intercepted arc in terms of  $\pi$  in a circle of diameter 60 in.

9.  $10^\circ$

10.  $60^\circ$

11.  $42^\circ$

12.  $50^\circ$

13.  $72^\circ$

14.  $110^\circ$

15.  $35^\circ$

16.  $65^\circ$

Given the measure of an arc, find the degree measure to the nearest tenth of the central angle it subtends in a circle of radius 16 cm.

17. 87 cm

18. 5.6 cm

19. 12 cm

20. 25 cm

21. 10.24 cm

22. 7.9 cm

23. 11 cm

24. 6 cm

Find the area of each sector to the nearest tenth, given its central angle,  $\theta$ , and the radius of the circle.

25.  $\theta = \frac{\pi}{6}$ ,  $r = 14$  cm

26.  $\theta = \frac{\pi}{6}$ ,  $r = 12$  ft

# Practice Worksheet

## Circular Functions

Find the values of the six trigonometric functions of an angle in standard position if the given point lies on its terminal side.

1.  $(-1, 5)$

2.  $(6, -8)$

3.  $(3, 2)$

4.  $(-3, -4)$

5.  $(0, -4)$

6.  $(7, 0)$

7.  $(\sqrt{2}, -\sqrt{2})$

8.  $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Suppose  $\theta$  is an angle in standard position whose terminal side lies in the given quadrant. For each function, find the values of the remaining five trigonometric functions of  $\theta$ .

9.  $\cos \theta = \frac{3}{5}$ ; quadrant I

10.  $\sin \theta = -\frac{2}{3}$ ; quadrant IV

**Practice Worksheet*****Trigonometric Functions of Special Angles***

*Find each exact value. Do not use a calculator.*

1.  $\sin \frac{\pi}{4}$

2.  $\cos \frac{\pi}{4}$

3.  $\tan \frac{\pi}{4}$

4.  $\cos 210^\circ$

5.  $\sin 300^\circ$

6.  $\tan 330^\circ$

7.  $\sin \frac{3\pi}{4}$

8.  $\cos \frac{3\pi}{4}$

9.  $\tan \frac{3\pi}{4}$

10.  $\sin 90^\circ$

11.  $\csc 270^\circ$

12.  $\tan 45^\circ$

13.  $\cos \frac{3\pi}{2}$

14.  $\tan \frac{3\pi}{2}$

15.  $\sin \frac{3\pi}{2}$

*Use a calculator to approximate each value to four decimal places.*

16.  $\cot(-75^\circ)$

17.  $\sin 634^\circ$

18.  $\cos 235^\circ$

19.  $\sin 2$

20.  $\sec 4.28$

21.  $\cot 0.23$

# Practice Worksheet

## Right Triangles

Solve each triangle described, given the triangle below. Round angle measures to the nearest degree and side measures to the nearest tenth.

1.  $A = 39^\circ 12'$ ,  $b = 2.1$

2.  $a = 9$ ,  $B = 49^\circ$

3.  $B = 64^\circ$ ,  $b = 19.2$

5.  $A = 16^\circ$ ,  $c = 14$

7.  $c = 21.3$ ,  $A = 26^\circ 20'$

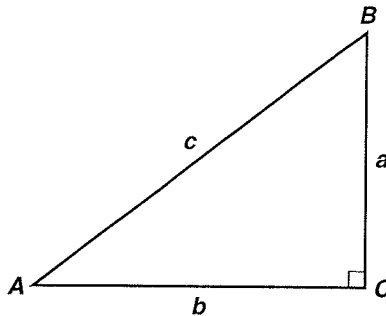
9.  $A = 55^\circ 55'$ ,  $c = 16$

4.  $B = 56^\circ 48'$ ,  $c = 63.1$

6.  $a = 0.4$ ,  $c = 0.5$

8.  $a = 2$ ,  $b = 7$

10.  $a = \sqrt{15}$ ,  $B = 18^\circ$



# Practice Worksheet

## Graphs of the Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1.  $\sin(-720^\circ)$

2.  $\tan(-180^\circ)$

3.  $\cos(540^\circ)$

4.  $\tan(180^\circ)$

5.  $\csc(720^\circ)$

6.  $\sec(180^\circ)$

Find the values of  $\theta$  for which each equation is true.

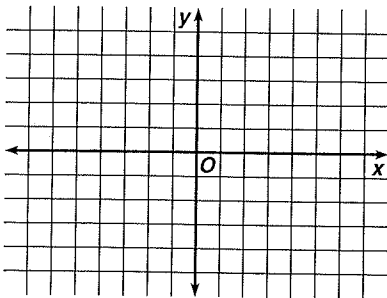
7.  $\sin \theta = -1$

8.  $\sec \theta = -1$

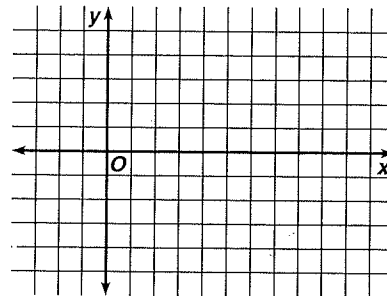
9.  $\tan \theta = 0$

Graph each function on the given interval.

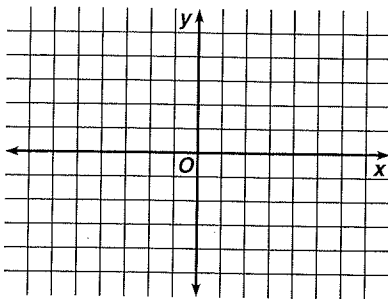
10.  $y = \sin x; -90^\circ \leq x \leq 90^\circ$



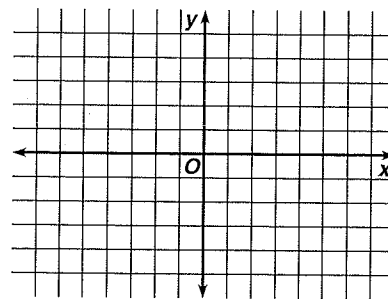
11.  $y = \tan x; -90^\circ \leq x \leq 270^\circ$



12.  $y = \cos x; -360^\circ \leq x \leq 360^\circ$



13.  $y = \sec x; -360^\circ \leq x \leq 360^\circ$



# Practice Worksheet

**Amplitude, Period, and Phase Shift** and Horizontal Shift.

State the amplitude, period, and phase shift and horizontal shift

1.  $y = -2 \sin \theta$

2.  $y = 10 \sec \theta$

3.  $y = -3 \sin 4\theta$

4.  $y = 0.5 \sin \left( \theta - \frac{\pi}{3} \right)$

5.  $y = 2.5 \cos (\theta + 180^\circ)$

6.  $y = -1.5 \sin \left( 4\theta - \frac{\pi}{4} \right)$

Write an equation of the sine function with each amplitude, period, and <sup>horizontal</sup> shift.

7. amplitude = 0.75, period =  $360^\circ$ , <sup>horizontal</sup> shift =  $30^\circ$

8. amplitude = 4, period =  $3^\circ$ , <sup>horizontal</sup> shift =  $-30^\circ$

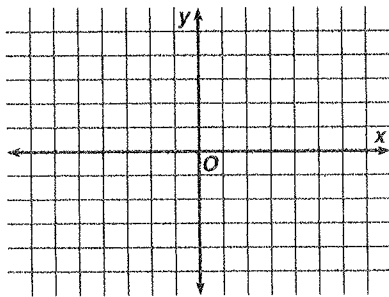
Write an equation of the cosine function with each amplitude, period, and phase shift.

9. amplitude = 3.75, period =  $90^\circ$ , phase shift =  $4^\circ$

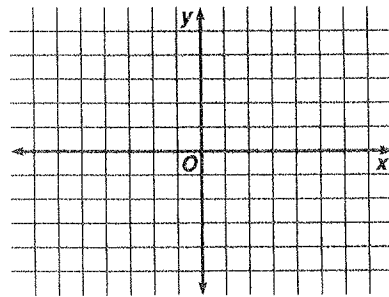
10. amplitude = 12, period =  $45^\circ$ , phase shift =  $180^\circ$

Graph each function.

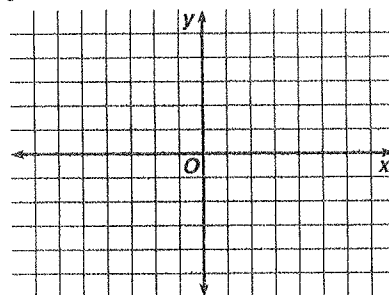
11.  $y = 0.5 \sin x$



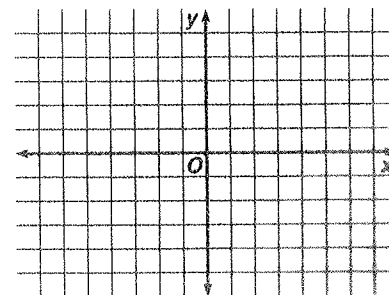
12.  $y = 2 \cos (3x)$



13.  $y = 2 \cos (2x - 45^\circ)$



14.  $y = \tan (x + 60^\circ)$

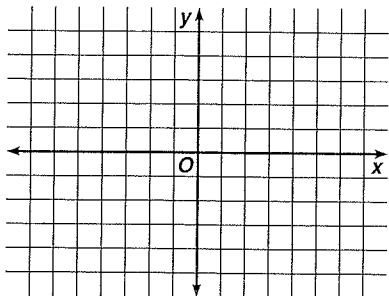


# Practice Worksheet

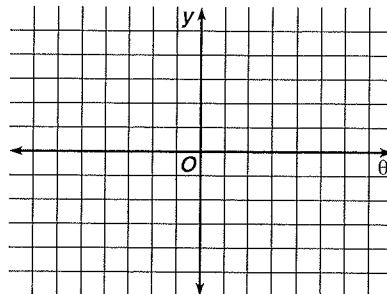
## Graphing Trigonometric Functions

Graph each function.

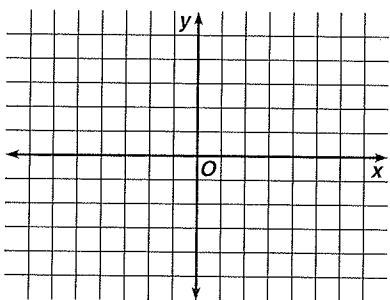
1.  $y = 2 \sin(x - 45^\circ)$



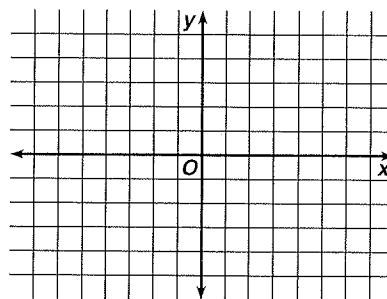
2.  $y = -2 \cos(3\theta)$



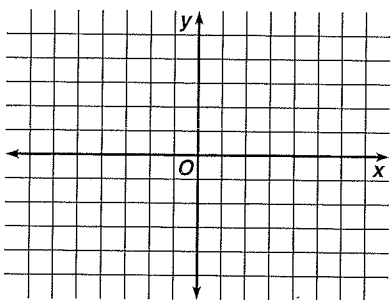
3.  $y = \frac{1}{2} \cos\left(x - \frac{\pi}{2}\right)$



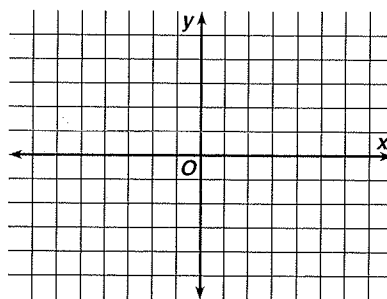
4.  $y = \sin\left(\frac{x}{2} + 90^\circ\right)$



5.  $y = \sin x + \cos x$



6.  $y = \cos 2x - \cos x$



# Practice Worksheet

## Inverse Trigonometric Functions

Write each equation in the form of an inverse relation.

1.  $0.75 = \sin x$

2.  $-1 = \cos x$

3.  $0.1 = \tan \theta$

4.  $\frac{3}{5} = \cos x$

5.  $\sin x = \frac{\sqrt{3}}{2}$

6.  $\cos \alpha = \frac{12}{13}$

Find the values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$  that satisfy each equation.

7.  $x = \arccos 1$

8.  $\arccos \frac{\sqrt{2}}{2} = x$

9.  $\arcsin \frac{1}{2} = x$

10.  $\sin^{-1}(-1) = x$

11.  $\sin^{-1} \frac{\sqrt{2}}{2} = x$

12.  $\cot^{-1} 1 = x$

Evaluate each expression. Assume that all angles are in Quadrant I.

13.  $\cos\left(\cos^{-1} \frac{1}{2}\right)$

14.  $\sin\left(\cos^{-1} \frac{1}{2}\right)$

15.  $\cos\left(\sin^{-1} \frac{1}{2}\right)$

16.  $\tan\left(\sin^{-1} \frac{\sqrt{2}}{2} - \cos^{-1} \frac{\sqrt{2}}{2}\right)$

17. Verify that  $\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = 90^\circ$ . Assume that all angles are in Quadrant I.

# Practice Worksheet

## *Principal Values of the Inverse Trigonometric Functions*

Find each value.

1.  $\text{Arcsin } (-1)$

2.  $\text{Arccos } 1$

3.  $\text{Arctan } (-1)$

4.  $\text{Cos}^{-1} \frac{1}{2}$

5.  $\text{Arcsin } 1$

6.  $\text{Tan}^{-1} \left( -\frac{\sqrt{3}}{3} \right)$

7.  $\cos \left( \text{Cos}^{-1} \left( -\frac{1}{2} \right) \right)$

8.  $\sin \left( \text{Sin}^{-1} \frac{\sqrt{3}}{2} \right)$

9.  $\tan \left( \text{Tan}^{-1} \frac{\sqrt{3}}{3} \right)$

10.  $\text{Cos}^{-1} \left( \text{Cos} \frac{\pi}{2} \right)$

11.  $\text{Sin}^{-1} \left( \sin \frac{\pi}{4} \right)$

12.  $\text{Tan}^{-1} \left( \tan \frac{\pi}{3} \right)$

13.  $\cos \left( \text{Arcsin} \frac{1}{2} \right)$

14.  $\sin \left( \text{Arccos} \frac{\sqrt{3}}{2} \right)$

15.  $\tan \left( \text{Arcsin} \frac{\sqrt{3}}{3} \right)$

16.  $\tan \left( \frac{1}{2} \text{Arccos} \frac{5}{13} \right)$

17.  $\cos \left( \frac{1}{2} \text{Arcsin} \frac{6}{10} \right)$

18.  $\sin \left( 2 \text{Arccos} \frac{3}{5} \right)$

19.  $\sin \left[ \text{Cos}^{-1} \left( \frac{\sqrt{2}}{2} \right) - \frac{\pi}{4} \right]$

20.  $\cos \left[ \text{Sin}^{-1} \left( \frac{\sqrt{2}}{2} \right) + \frac{\pi}{4} \right]$

21.  $\text{Tan} \left[ \frac{3\pi}{4} + \text{Sin}^{-1} \frac{\sqrt{2}}{2} \right]$

# Practice Worksheet

## Graphing Inverses of Trigonometric Functions

State the domain and range of each relation.

1.  $y = \sin x + 1$

2.  $y = \sin x + 1$

3.  $y = \cos x - 1$

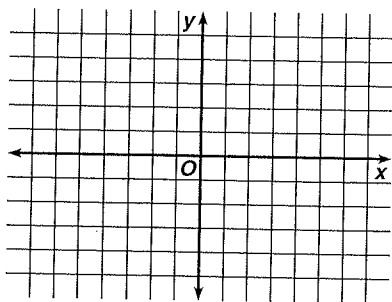
4.  $y = \cos^{-1} x$

5.  $y = \arcsin x$

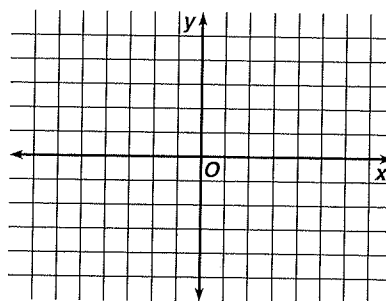
6.  $y = \tan^{-1} x$

Write the equation for the inverse of each function. Then graph the function and its inverse.

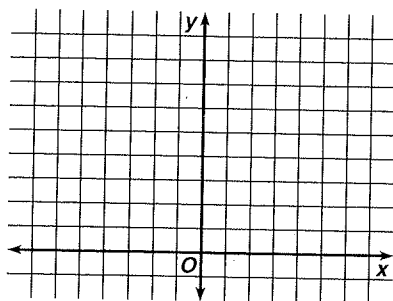
7.  $y = \cos^{-1} x$



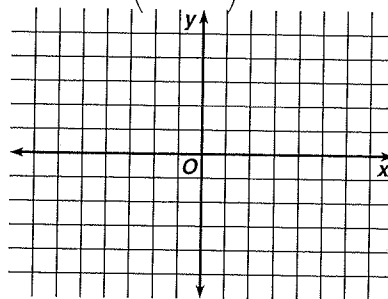
8.  $y = \tan^{-1}(3x)$



9.  $y = \frac{\pi}{2} + \cos^{-1} x$



10.  $y = \sin\left(x - \frac{\pi}{2}\right)$



Determine if each of the following is true or false. If false, give a counterexample.

11.  $\cos^{-1} x = \cos^{-1}(-x)$

12.  $\sin^{-1} x = -\sin^{-1} x$

# Practice Worksheet

## Basic Trigonometric Identities

Solve for values of  $\theta$  between  $0^\circ$  and  $90^\circ$ .

*Use Pythagorean Identities  
(not triangles)*

1. If  $\tan \theta = 2$ , find  $\cot \theta$ .

2. If  $\sin \theta = \frac{2}{3}$ , find  $\cos \theta$ .

3. If  $\cos \theta = \frac{1}{4}$ , find  $\tan \theta$

4. If  $\tan \theta = 3$ , find  $\sec \theta$ .

5. If  $\sin \theta = \frac{7}{10}$ , find  $\cot \theta$ .

6. If  $\tan \theta = \frac{7}{2}$ , find  $\sin \theta$ .

Express each value as a function of an angle in Quadrant I.

7.  $\sin 458^\circ$

8.  $\cos 892^\circ$

9.  $\tan (-876^\circ)$

10.  $\csc 495^\circ$

Simplify.

11.  $\frac{\cot A}{\tan A}$

12.  $\frac{\sin^2 \beta \cot \beta}{\cos \beta}$

13.  $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$

14.  $\cos x + \sin x \tan x$

15.  $\frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta}$

16.  $\frac{\cos^2 \theta}{1 + \sin \theta}$

\*17.  $\frac{\tan^2 \theta - 1}{\tan^2 \theta + 1} + 2 \cos^2 \theta$

\*18.  $1 - \frac{\sin^2 \theta}{1 + \cos \theta}$

# Practice Worksheet

## Verifying Trigonometric Identities

Verify that each of the following is an identity.

1.  $\frac{\csc x}{\cot x + \tan x} = \cos x$

2.  $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

3.  $\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2\sec^2 y$

4.  $1 - 2 \sin^2 r + \sin^4 r = \cos^4 r$

5.  $\tan u + \frac{\cos u}{1 + \sin u} = \sec u$

6.  $\frac{\tan x + \sec x}{\sec x - \cos x + \tan x} = \csc x$

Find a numerical value of one trigonometric function of each  $x$ .

7.  $\sin x = 3 \cos x$

8.  $\cos x = \cot x$