Name:

Precalculus

Trigonometric Functions

2.FMC

I. AMC (Merrill) Practice Worksheets
II. FMC (Connally) Exercises & Problems
III. PTCA (Foerster) Explorations
IV. Other Worksheets
V. Formulas and notes

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FMC 6.1 A

Exercises and Problems for Section 6.1

Exercises

In Exercises 1–8, do the functions appear to be periodic with period less than 4?

1.

t	0	1	2	3	4	5	6
$f(t)$	1	5	7	1	5	7	1

2.

r	0	π	2π	3π	4π	5π	6π	7π
$q(r)$	0	1	0	-1	1	0	1	0

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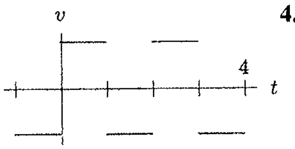

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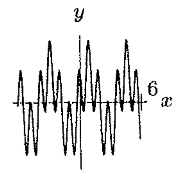

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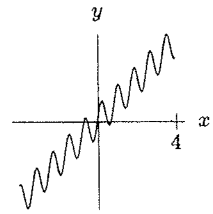

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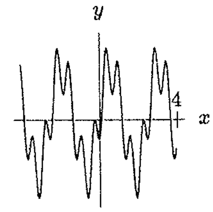

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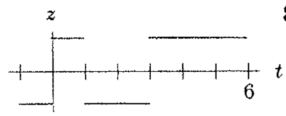

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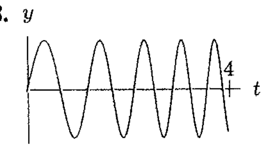

In Exercises 9–12, estimate the period of the periodic functions.

9.

t	0	1	2	3	4	5	6
$f(t)$	12	13	14	12	13	14	12

10.

z	1	11	21	31	41	51	61	71	81
$g(z)$	5	3	2	3	5	3	2	3	5

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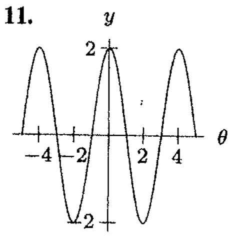

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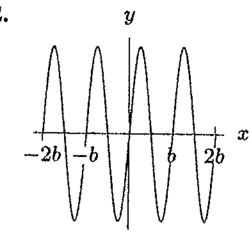


30. Table 6.3 gives the height $h = f(t)$ in feet of a weight on a spring where t is time in seconds. Find the midline, amplitude and period of the function f.

MAKE A GRAPH

Table 6.3

t	0	1	2	3	4	5	6	7
h	4.0	5.2	6.2	6.5	6.2	5.2	4.0	2.8
t	8	9	10	11	12	13	14	15
h	1.8	1.5	1.8	2.8	4.0	5.2	6.2	6.5

 FMC 6.1B

Problems 23–26 concern a weight suspended from the ceiling by a spring. (See Figure 6.8.) Let d be the distance in centimeters from the ceiling to the weight. When the weight is motionless, $d = 10$. If the weight is disturbed, it begins to bob up and down, or *oscillate*. Then d is a periodic function of t, time in seconds, so $d = f(t)$.

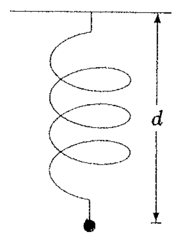


Figure 6.8

23. Determine the midline, period, amplitude, and the minimum and maximum values of f from the graph in Figure 6.9. Interpret these quantities physically; that is, use them to describe the motion of the weight.

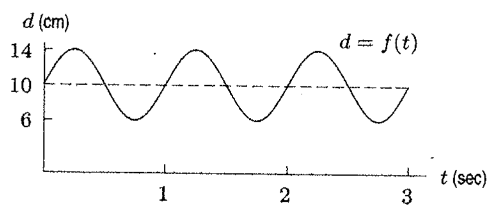


Figure 6.9

24. A new experiment with the same weight and spring is represented by Figure 6.10. Compare Figure 6.10 to Figure 6.9. How do the oscillations differ? For both figures, the weight was disturbed at time $t = -0.25$ and then left to move naturally; determine the nature of the initial disturbances.

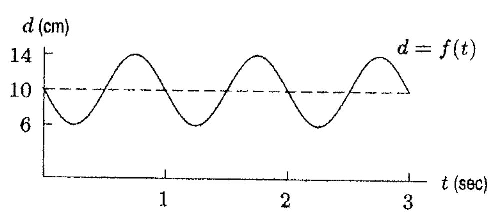


Figure 6.10

25. The weight in Problem 23 is gently pulled down to a distance of 14 cm from the ceiling and released at time $t = 0$. Sketch its motion for $0 \le t \le 3$.

26. Figures 6.11 and 6.12 describe the motion of two different weights, A and B, attached to two different springs. Based on these graphs, which weight:

(a) Is closest to the ceiling when not in motion?
(b) Makes the largest oscillations?
(c) Makes the fastest oscillations?

NOTE VERTICAL SCALES

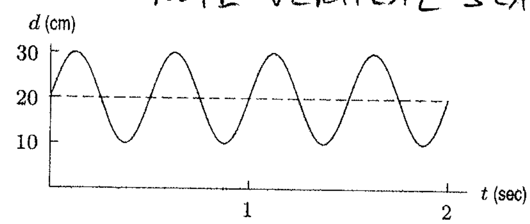


Figure 6.11: Weight A

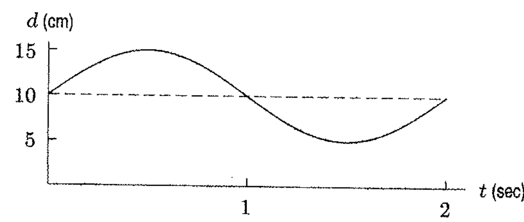
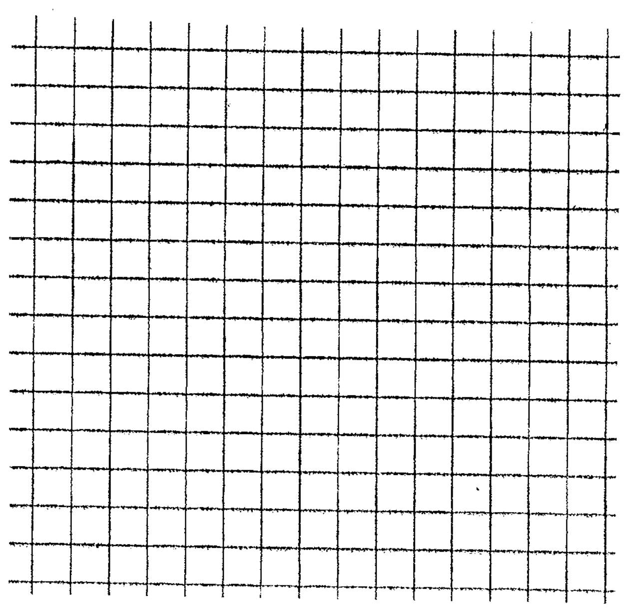


Figure 6.12: Weight B



FMC 6.2A

28. Find an angle ϕ, with $0° < \phi < 360°$, that has the same
(a) Cosine as $53°$ (b) Sine as $53°$

30. A revolving door (which rotates counterclockwise in Figure 6.28) was designed with five equally spaced panels for the entrance to the Pentagon. The arcs BC and AD have equal length.

(a) What is the angle between two adjacent panels?

(b) A four-star general enters by pushing on the panel at point B, and leaves the panel at point D. What is the angle of rotation?

(c) With the door in the position shown in Figure 6.28, an admiral leaves the Pentagon by pushing the panel between A and D to point B. What is the angle of rotation?

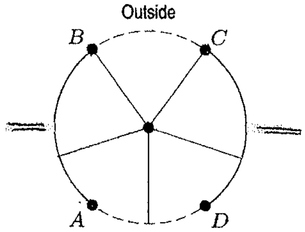


Figure 6.28

32. Let θ be an angle in the first quadrant, and suppose $\sin\theta = a$. Evaluate the following expressions in terms of a. (See Figure 6.30.)

(a) $\sin(\theta + 360°)$ **(b)** $\sin(\theta + 180°)$
(c) $\cos(90° - \theta)$ **(d)** $\sin(180° - \theta)$
(e) $\sin(360° - \theta)$ **(f)** $\cos(270° - \theta)$

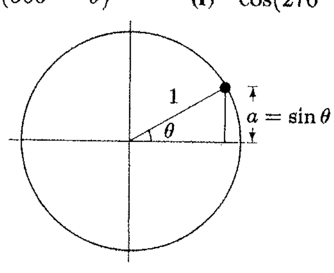


Figure 6.30

31. For the angle ϕ shown in Figure 6.29, sketch each of the following angles.

(a) $180 + \phi$ **(b)** $180 - \phi$ **(c)** $90 - \phi$ **(d)** $360 - \phi$

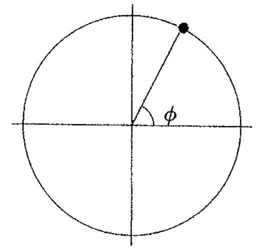


Figure 6.29

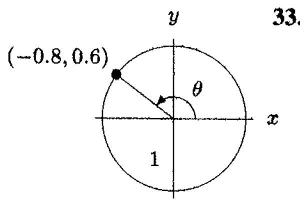
FMC 6.3A

29. Without using a calculator, give the sign of each of the following numbers:

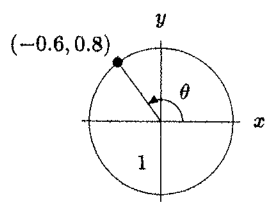
(a) $\cos 3$ **(b)** $\sin 4$ **(c)** $\sin(-4)$ **(d)** $\cos 7$

Evaluate $\sin\theta$ and $\cos\theta$ for the angle θ on the unit circle in Problems 32–33.

32.



33.



34. An ant starts at the point $(1, 0)$ on the unit circle and walks counterclockwise a distance of 3 units around the circle. Find the x and y coordinates (accurate to 2 decimal places) of the final location of the ant.

36. For ϕ in Figure 6.43, sketch the following angles.

(a) $\pi + \phi$ **(b)** $\pi - \phi$
(c) $\pi/2 - \phi$ **(d)** $2\pi - \phi$

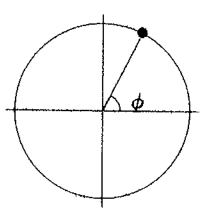


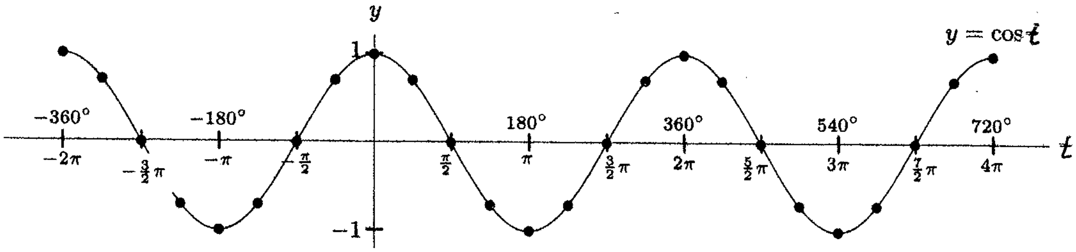
Figure 6.43

38. How far does the tip of the minute hand of a clock move in 35 minutes if the hand is 6 inches long?

44. a) Do you think there is a value of t for which $\cos t = t$? If so, estimate the value of t. If not, explain why not.
USE RADIANS.

44b, What about $f(t) = \frac{\sin(t)}{t}$?
- does this have a value at $t=0$?
- What about "near" $t=0$?

FMC 6.4A



27. **(a)** Match the lengths p, q, r, s marked on the unit circle in Figure 6.55 with the following values:

(i) $t = 0.8$ (ii) $t = \pi - 2.9$
(iii) $\cos(0.8)$ (iv) $-\cos(2.9)$

(b) On a graph of $y = \cos t$, sketch segments corresponding to each of the values in part (a).

graph

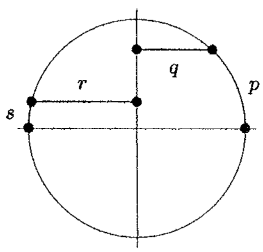


Figure 6.55

16. Match each of the letters A–G in Figure 6.49 to one of the following values of x (in radians): $1, 2, 4, 5, \pi/2, \pi$, and $3\pi/2$.

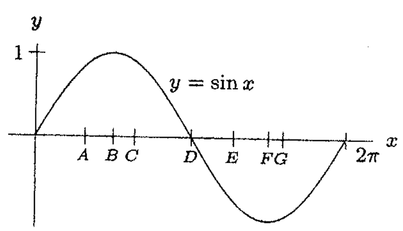


Figure 6.49

20. Figure 6.53 shows $y = \sin(x - \frac{\pi}{2})$ and $y = \sin(x + \frac{\pi}{2})$ starting at $x = 0$. Identify which is which.

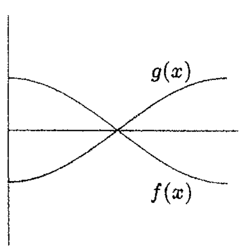


Figure 6.53

28. **(a)** Write an expression for the slope of the line segment joining P and Q in Figure 6.56.

(b) Evaluate your expression for $a = \pi/4$, $b = 4\pi/3$. Give an exact value for your answer.

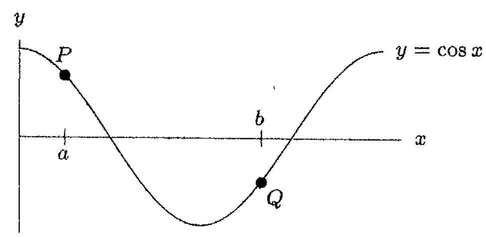


Figure 6.56

30. A circle of radius 5 is centered at the point $(-6, 7)$. Find a formula for $f(\theta)$, the x-coordinate of the point P in Figure 6.58.

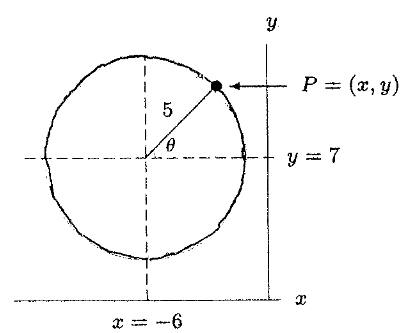


Figure 6.58

FMC 6.5A

In Exercises 1–4, state the period, amplitude, and midline.

Also state the horizontal shift and phase shift. (see below)

1. $y = 7\sin(4(t+7)) - 8$
2. $y = 6\sin(t+4)$
3. $y = \pi\cos(2t+4) - 1$
4. $2y = \cos(8(t-6)) + 2$

32. Find a possible formula for the trigonometric function whose values are in the following table.

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$g(x)$	2	2.6	3	3	2.6	2	1.4	1	1	1.4	2

38. The pressure, P (in lbs/ft^2), in a pipe varies over time. Five times an hour, the pressure oscillates from a low of 90 to a high of 230 and then back to a low 90. The pressure at $t = 0$ is 90.

(a) Graph $P = f(t)$, where t is time in minutes. Label your axes.
(b) Find a possible formula for $P = f(t)$.
(c) By graphing $P = f(t)$ for $0 \le t \le 2$, estimate when the pressure first equals 115 lbs/ft^2.

In Exercises 13–20, find formulas for the trigonometric functions.

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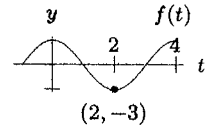

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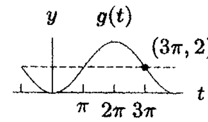

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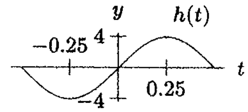

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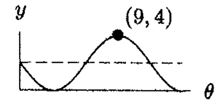

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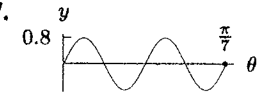

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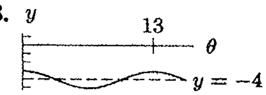

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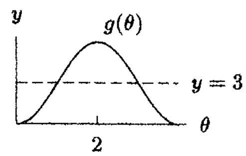

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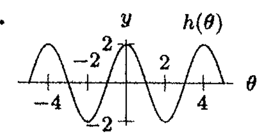


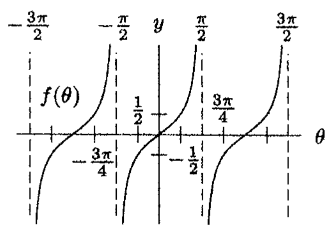
$$y = A\sin(B(t-h)) + k,$$
$$y = A\cos(B(t-h)) + k.$$

Amplitude $= |A|$; period $= 2\pi/|B|$; midline $y = k$.
Phase shift $= Bh$; horizontal shift $= h$.

FMC 6.6A

In Exercises 16–17, give a possible formula for the function.

16.



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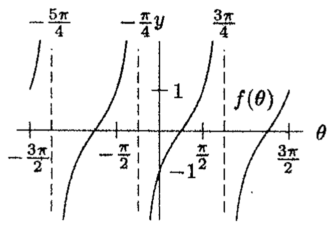

22. **(a)** $\cos\alpha = -\sqrt{3}/5$ and α is in the third quadrant. Find exact values for $\sin\alpha$ and $\tan\alpha$.
(b) $\tan\beta = 4/3$ and β is in the third quadrant. Find exact values for $\sin\beta$ and $\cos\beta$.

Problems 26–29 give an expression for one of the three functions $\sin\theta$, $\cos\theta$, or $\tan\theta$, with θ in the first quadrant. Find expressions for the other two functions. Your answers will be algebraic expressions in terms of x.

26. $\sin\theta = x/3$

27. $\cos\theta = 4/x$

28. $x = 2\cos\theta$

29. $x = 9\tan\theta$

30. **(a)** Find an equation for the line l in Figure 6.76.
(b) Find the x-intercept of the line.

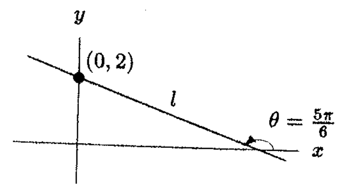

Figure 6.76

Find exact values for the lengths of the labeled segments in Problems 34–35.

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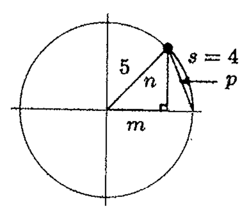

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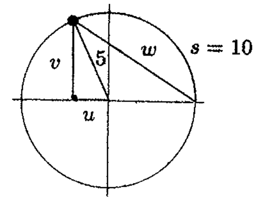


FMC 6.7A

36. In your own words, explain what each of the following expressions means. Evaluate each expression for $x = 0.5$. Give an exact answer if possible.

(a) $\sin^{-1} x$ **(b)** $\sin(x^{-1})$ **(c)** $(\sin x)^{-1}$.

Solve the equations in Problems 29–32 for $0 \le t \le 2\pi$. First estimate answers from a graph; then find exact answers.

29. $\cos(2t) = \dfrac{1}{2}$

30. $\tan t = \dfrac{1}{\tan t}$

31. $2 \sin t \cos t - \cos t = 0$

32. $3 \cos^2 t = \sin^2 t$

35. Approximate the x-coordinates of points P and Q shown in Figure 6.90, assuming that the curve is a sine curve. [Hint: Find a formula for the curve.]

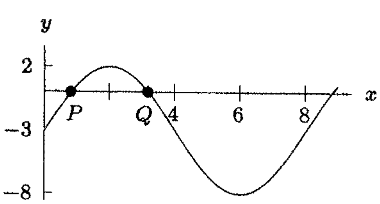


Figure 6.90

44. You are perched in the crow's nest, C, on top of the mast of a ship, S. See Figure 6.91. You will calculate how far you can see when you are x meters above the surface of the ocean.

(a) Find formulas for d, the distance you can see to the horizon, H, and l, the distance to the horizon along the earth's surface, in terms of x, the height of the ship's mast, and r, the radius of the earth.

(b) How far is the horizon from the top of a 50-meter mast? How far, measured along the earth's surface, is the horizon from the ship's position on the ocean? Use $r = 6{,}370{,}000$ meters.

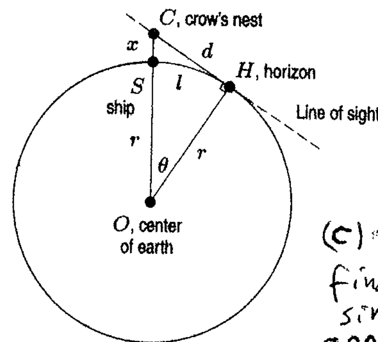


Figure 6.91

(c) Try to find a simple approximate formula

45. Let k be a positive constant and t be an angle measured in radians. Consider the equation

$$k \sin t = t^2.$$

(a) Explain why any solution to the equation must be between $-\sqrt{k}$ and $\sqrt{k}$, inclusive.

(b) Approximate every solution to the equation when $k = 2$.

(c) Explain why the equation has more solutions for larger values of k than it does for small values.

(d) Approximate the least value of k, if any, for which the equation has a negative solution.

FMC7.5A

Convert the polar coordinates in Exercises 14–17 to Cartesian coordinates. Give exact answers.

14. $(1, 2\pi/3)$

15. $(\sqrt{3}, -3\pi/4)$

16. $(2\sqrt{3}, -\pi/6)$

17. $(2, 5\pi/6)$

Convert the Cartesian coordinates in Problems 18–21 to polar coordinates.

18. $(1, 1)$

19. $(-1, 0)$

20. $(\sqrt{6}, -\sqrt{2})$

21. $(-\sqrt{3}, 1)$

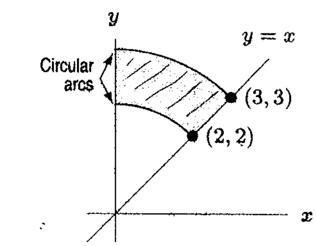
For Problems 22–28, the origin is at the center of a clock, with the positive x-axis going through 3 and the positive y-axis going through 12. The hour hand is 3 cm long and the minute hand is 4 cm long. What are the Cartesian coordinates and polar coordinates of the tips of the hour hand and minute hand, H and M, respectively, at the following times?

22. 12 noon **23.** 3 pm **24.** 9 am **25.** 1:30 pm

26. 7 am **27.** 3:30 pm **28.** 9:15 am

In Problems 29–31, give inequalities for r and θ which describe the following regions in polar coordinates.

29.



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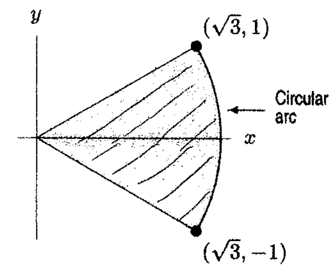
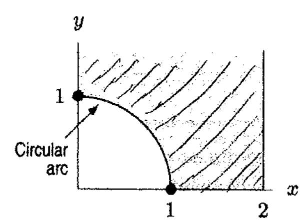


31.



Note: Region extends indefinitely in the y-direction.