Name: ______________________

Precalculus

Trigonometric Functions

3. PTCA

I. AMC (Merrill) Practice Worksheets
II. FMC (Connally) Exercises & Problems
III. PTCA (Foerster) Explorations
IV. Other Worksheets
V. Formulas and notes

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v-2.0 2007-2008

Name: ____________ Group Members: ____________

Exploration 2-1a: Transformed Periodic Functions

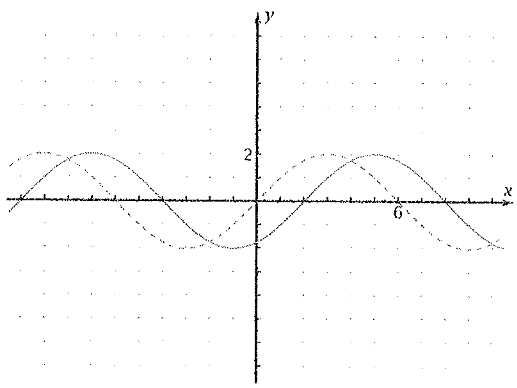
Date: ____________

Objective: Given a pre-image graph and a transformed graph of a periodic function, state the transformation(s).

Give the transformation applied to $f(x)$ (dashed) to get the solid graph, $y = g(x)$.

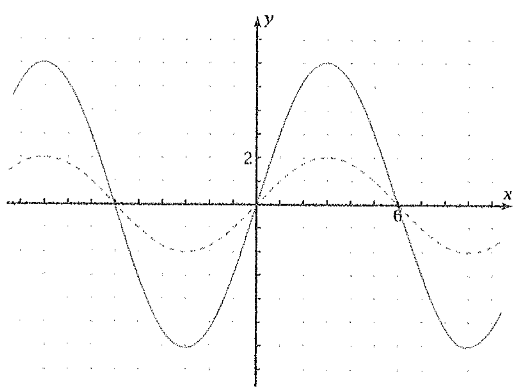
1. Verbally: ____________

   Equation: $y = g(x) =$ ____________



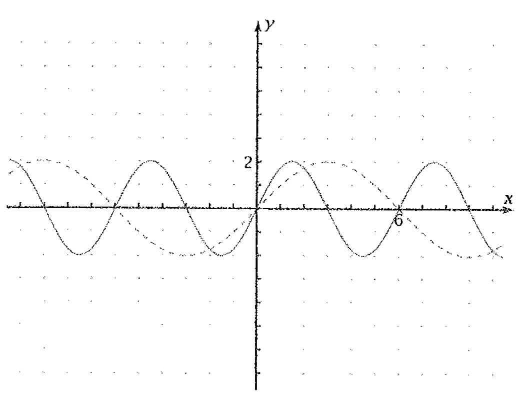
2. Verbally: ____________

   Equation: $y = g(x) =$ ____________



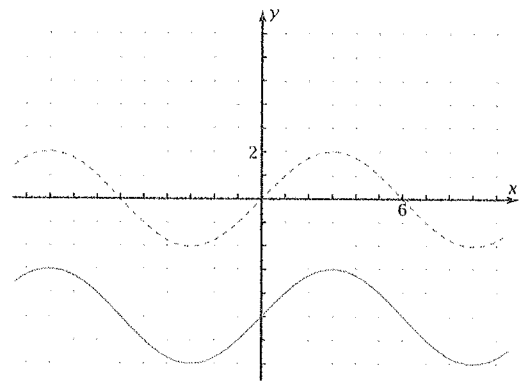
3. Verbally: ____________

   Equation: $y = g(x) =$ ____________



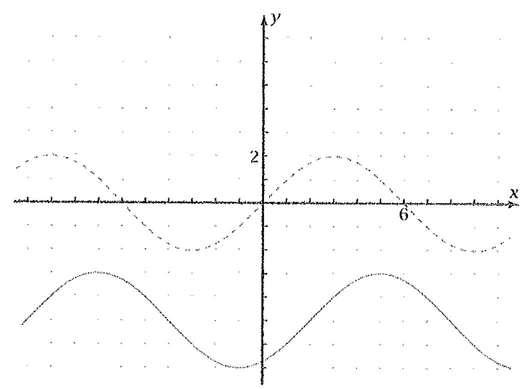
4. Verbally: ____________

   Equation: $y = g(x) =$ ____________



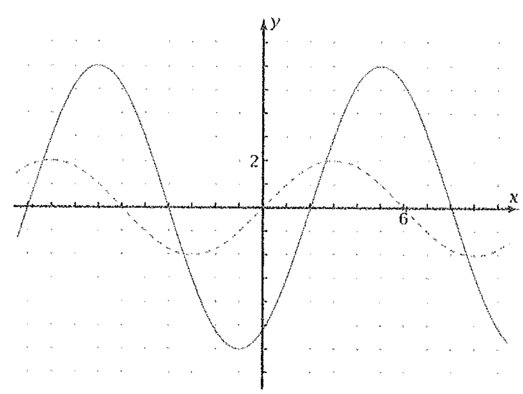
5. Verbally: ____________

   Equation: $y = g(x) =$ ____________



6. Verbally: ____________

   Equation: $y = g(x) =$ ____________



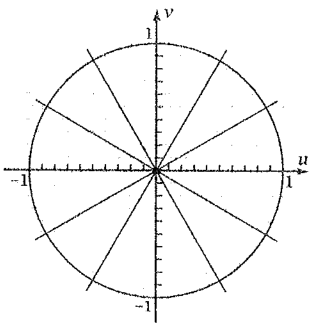
7. What did you learn as a result of doing this Exploration that you did not know before?

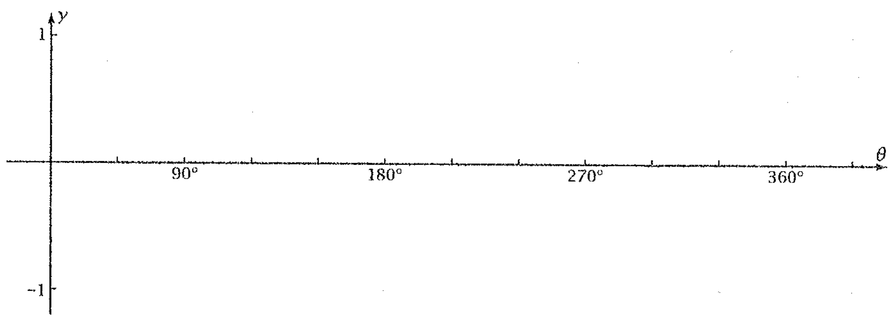
Name: ______________ Group Members: ______________

Exploration 2-3b: *uv*-Graphs and *θy*-Graphs of Sinusoids

Date: ______________

Objective: Show a geometric relationship between angles plotted as angles and angles plotted along the θ-axis.





1. The left figure shows a unit circle in a *uv*-diagram with angles marked at every 30°. Read, to two decimal places, the coordinates (u, v) of the point where the ray at 60° cuts the unit circle.

2. Find cos 60° and sin 60° with your calculator. Explain how these numbers relate to the answers to Problem 1.

3. Plot the point $(\theta, y) = (60°, \sin 60°)$ on the θy-coordinate system on the right at the top of this Exploration. Draw a line segment showing how this point is related to the point you plotted in Problem 1.

4. Without actually calculating any more values, plot points on the graph of $y = \sin\theta$ for each 30° from 0° to 360°. Show segments connecting the appropriate points on the *uv*-diagram with points in the θy-diagram.

5. Connect the points in Problem 4 with a smooth curve. What geometrical figure is this curve?

6. Use your observation in Problem 2 to plot points on the graph of $y = \cos\theta$ for each 30° from $\theta = 0°$ to $\theta = 360°$. Connect the points with a smooth curve.

7. What transformation could you apply to the graph of $y = \sin\theta$ to get the graph of $y = \cos\theta$?

8. Explain the difference between the way the value of θ appears on the *uv*-diagram and the way it appears on the θy-diagram.

9. Why do you think the letters *u* and *v*, rather than the more common letters *x* and *y*, are used in the figure on the left at the top of this Exploration?

10. What did you learn as a result of doing this Exploration that you did not know before?

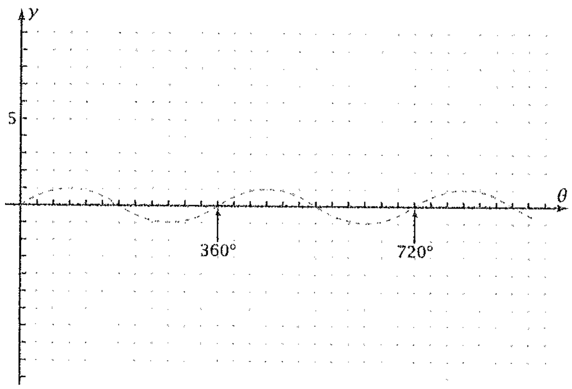
Name: ______________ Group Members: ______________

Exploration 2-3c: Parent Sinusoids

Date: ______________

Objective: Explore the graph of the parent function $y = \sin x$, and transform the graph.

1. The graph shows the function $y = \sin x$. Plot this graph as y_1 on your grapher. Use the window shown. Turn on the grid to get the dots. Does your graph agree with this figure? ______



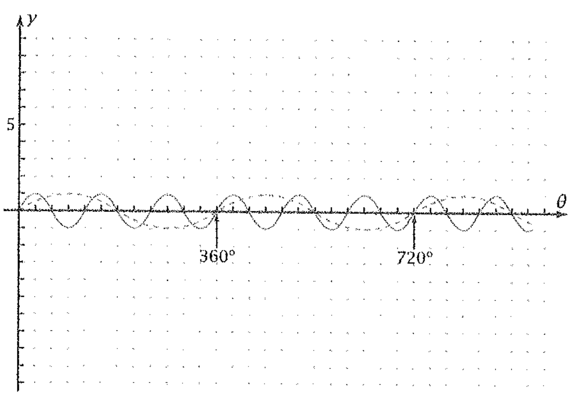
2. The **amplitude** of a periodic function is the vertical distance from the central axis to a high or low point. What is the amplitude of the sine function in Problem 1? Write the equation of the transformed function that would have an amplitude of 5.

3. Plot the transformed graph as y_2 on your grapher. Does the resulting graph really have an amplitude of 5? ______

4. The solid graph shows a transformation of the sine function from Problem 1. Identify the transformation, and write the equation for the transformed graph. Confirm that your answer is correct by plotting your equation as y_3.

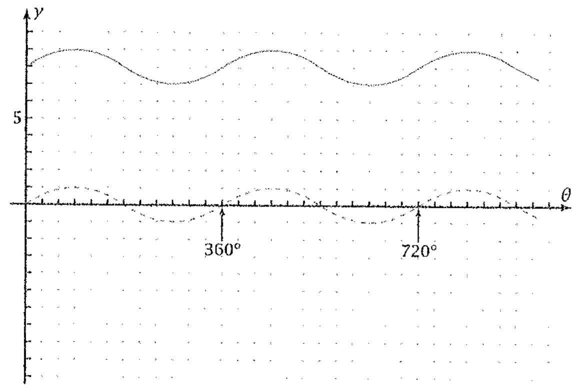
   Verbally: ______________

   Equation: ______________

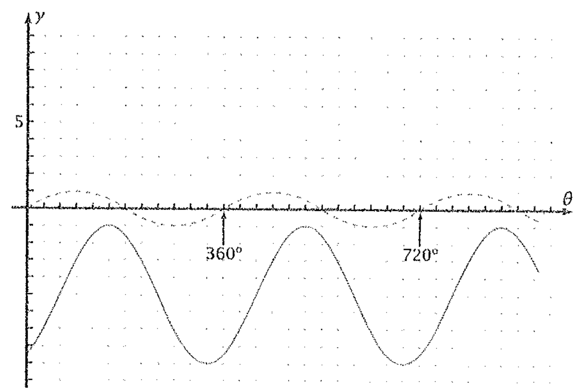


5. Write the equation for this transformed graph. Duplicate this graph on your grapher.

   Equation: ______________



6. The dotted graph shows the result of three transformations. State each transformation, write the equation of the transformed graph, and duplicate the graph on your grapher.



7. Degrees can be used to measure **rotation.** What do you think is the significance of the fact that the **period** of the sine function in Problem 1 is 360°?

8. What did you learn as a result of doing this Exploration that you did not know before?

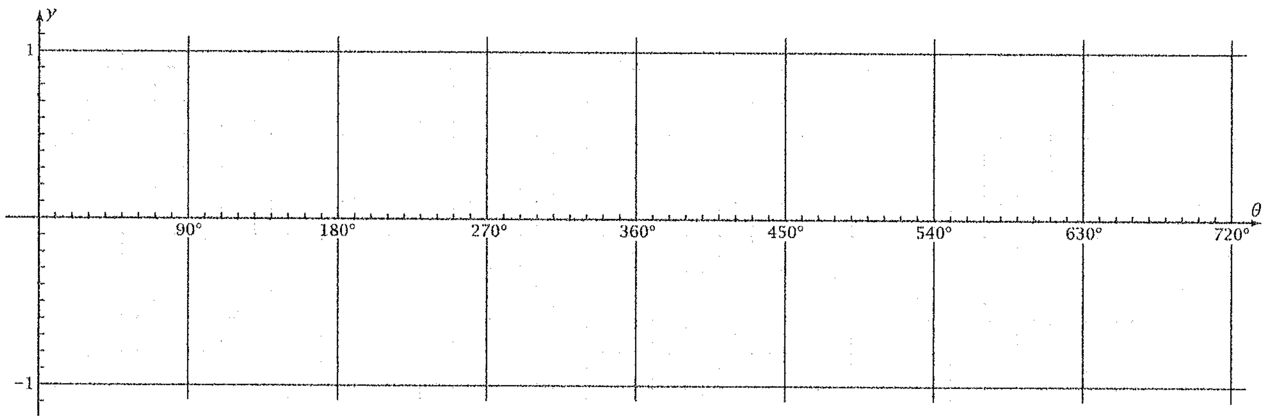
Name: ________________ Group Members: ________________________________

Exploration 3-1b: Sine and Cosine Graphs, Manually

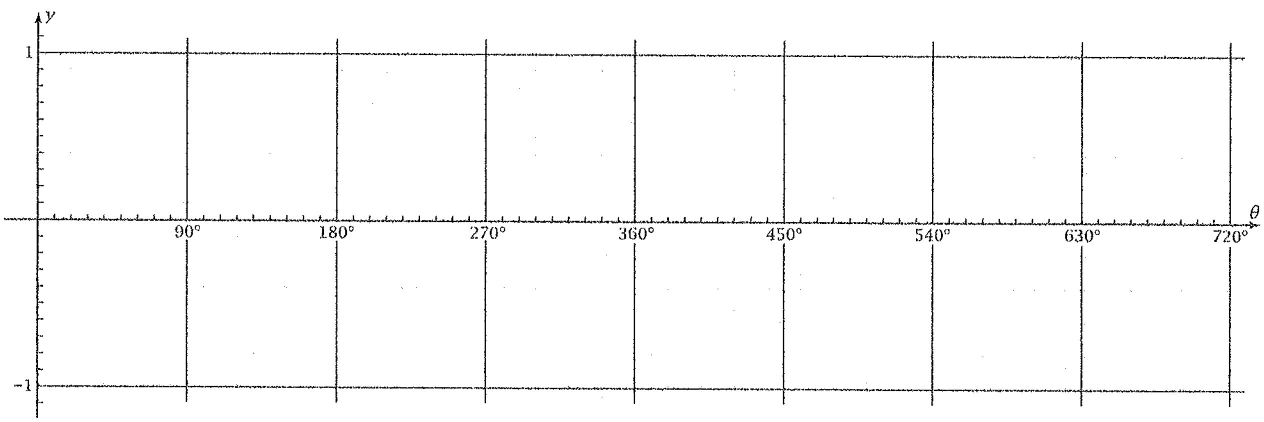
Date: ____________

Objective: Find the shape of sine and cosine graphs by plotting them on graph paper.

1. On your grapher, make a table of values of $y = \sin\theta$ for each 10° from 0° to 90°. Set the mode to round to 2 decimal places. Plot the values on this graph paper. Also plot $y = \sin\theta$ for each 90° through 720°. Connect the points with a smooth curve, observing the shape you plotted for 0° to 90°.



2. Plot the graph of $y = \cos\theta$ pointwise, the way you did for sine in Problem 1.



3. Find sin 45° and cos 65°. Show that the corresponding points are on the graphs in Problems 1 and 2, respectively.

4. Find the inverse trigonometric functions $\theta = \sin^{-1} 0.4$ and $\theta = \cos^{-1} 0.8$ Show that the corresponding points are on the graphs in Problems 1 and 2, respectively.

5. What are the ranges of the sine and cosine functions?

6. Name a real-world situation where variables are related by a periodic graph like sine or cosine.

7. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________ Group Members: ______________

Exploration 3-3a: Tangent and Secant Graphs

Date: ______________

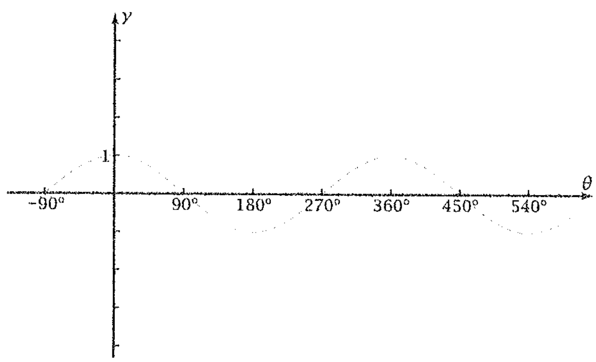
Objective: Discover what the tangent and secant function graphs look like and how they relate to sine and cosine.

No graphers allowed for Problems 1–7.

1. The **reciprocal property** states that

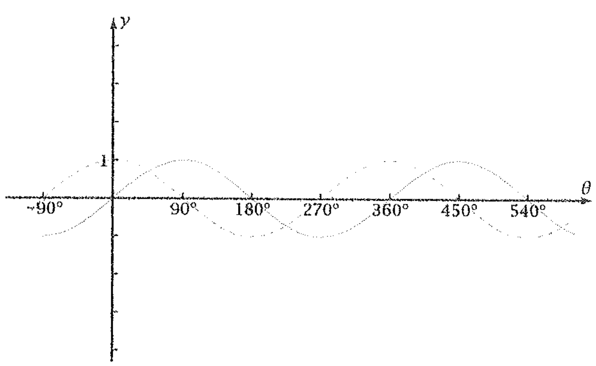
$$\sec\theta = \frac{1}{\cos\theta}$$

Without your grapher, use this property to sketch the graph of $y = \sec\theta$ on the same axes as the graph of the **parent function** $y = \cos\theta$. In particular, show what happens to the secant graph wherever $\cos\theta = 0$.



2. Write the **quotient property** expressing $\tan\theta$ as a quotient of two other trigonometric functions.

3. The next figure shows the parent functions $y = \sin\theta$ and $y = \cos\theta$. Based on the answer to Problem 2, determine where the asymptotes are for the graph of $y = \tan\theta$, and mark them on the figure.



4. Based on the quotient property, find out where the θ-intercepts are for the graph of $y = \tan\theta$. Mark these intercepts on the figure in Problem 3.

5. At $\theta = 45°$, $\sin\theta$ and $\cos\theta$ are equal. Based on this fact, what does tan 45° equal? Mark this point on the graph in Problem 3. Mark all other points where $|\sin\theta| = |\cos\theta|$.

   $\tan 45° =$ ______________

6. Use the points and asymptotes you have marked to sketch the graph of $y = \tan\theta$ on the figure in Problem 3. (No graphers allowed!)

7. Check your graphs with your instructor. ______________

Graphers allowed for the remaining problems.

8. On your grapher, plot the graph of $y = \csc\theta$. Sketch the result here.

9. On your grapher, plot the graph of $y = \cot\theta$. Sketch the result here.

10. At what values of θ are the points of inflection for $y = \tan\theta$? Explain why the tangent function has no critical points.

11. Explain why the graph of $y = \sec\theta$ has no points of inflection, even though the graph goes from concave up to concave down at various places.

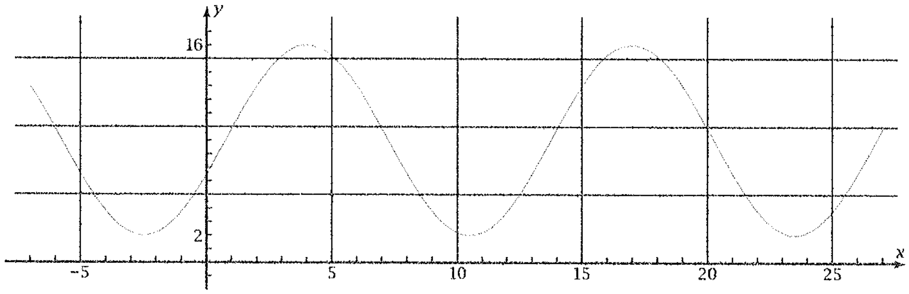
12. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________ Group Members: ______________

Exploration 3-6b: Given y, Find x Algebraically

Date: ______________

Objective: Given the particular equation for a sinusoid and a value of y, calculate the corresponding x-values algebraically.



1. The sinusoid has equation

$$y = 9 + 7\cos\frac{2\pi}{13}(x - 4)$$

Confirm that this equation gives the correct value of y when $x = 15$.

2. Your objective is to find algebraically the values of x given $y = 5$. Substitute 5 for y. Then do the algebra necessary to get x using an arccosine. Write the **general solution** in the form

$$x = (\text{number}) + (\text{period})n \text{ or } (\text{number}) + (\text{period})\,n$$

3. Write the two values of x from the general solution in the $n = 0$ row of this table. By adding and subtracting multiples of the period, fill in the other rows in the table with more possible values of x.

n	x_1	x_2
−1		
0		
1		
2		

4. Circle the points on the given graph where the line $y = 5$ cuts the graph. For each point, tell the value of n at that point.

5. Find the two values of x if $n = 100$.

6. Find the first value of x greater than 1000 for which $y = 5$. What does n equal there?

7. What did you learn as a result of doing this Exploration that you did not know before?

Name: ________________ Group Members: ________________________________

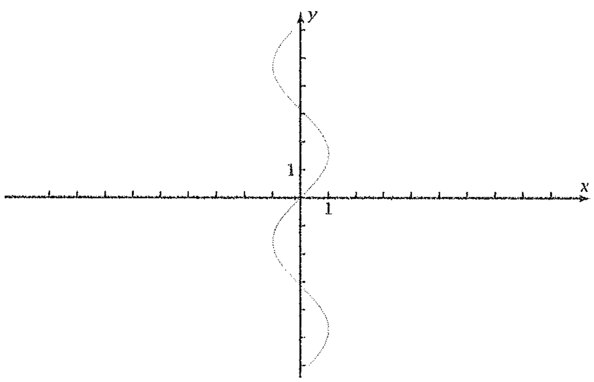
Exploration 4-6b: Principal Branches of Inverse Trigonometric Relations

Date: __________

Objective: Figure out the principal branches of each of the six inverse circular functions.

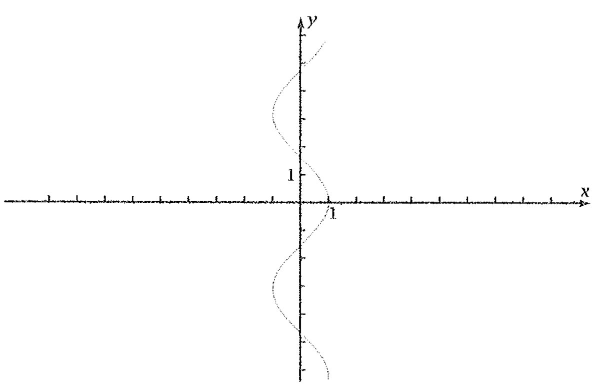
1. On your grapher, plot the inverse circular function $y = \sin^{-1} x$. Use equal scales on the two axes. On the graph of $y = \arcsin x$ shown here, darken the **principal branch** of the inverse sine relation on the part of the graph that is the inverse sine function. Give the range of the principal branch.

   Range: ______________________________



2. Use the technique in Problem 1 to find the range of $y = \cos^{-1} x$. Darken the principal branch on the graph of $y = \arccos x$, shown next.

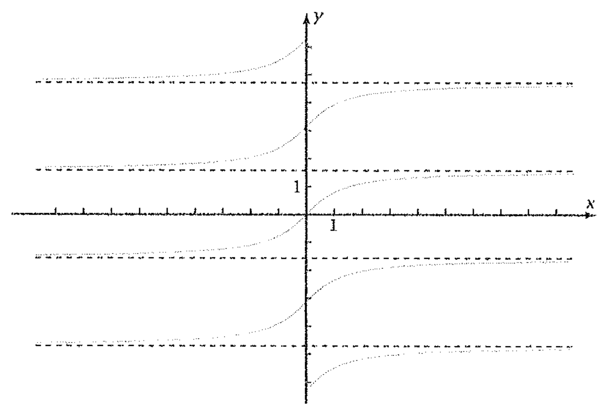
   Range: ______________________________



3. Why can't the range of the inverse cosine function ($y = \cos^{-1} x$) be the same as the range of the inverse sine function ($y = \sin^{-1} x$)?

4. Use the technique in Problems 1 and 2 to find the range of $y = \tan^{-1} x$. Darken the principal branch on this graph of $y = \arctan x$.

   Range: ______________________________



5. The range of $y = \sin^{-1} x$ is the closed interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Explain why the range of $y = \tan^{-1} x$ cannot include the endpoints.

6. If the range of the inverse tangent function ($y = \tan^{-1} x$) were $[0, \pi]$ (excluding $\frac{\pi}{2}$), like the range of $y = \cos^{-1} x$, then $y = \tan^{-1} x$ would still be a function. What disadvantage would there be to defining the range of $y = \tan^{-1} x$ this way?

(Over)

Name: ______________ Group Members: ______________

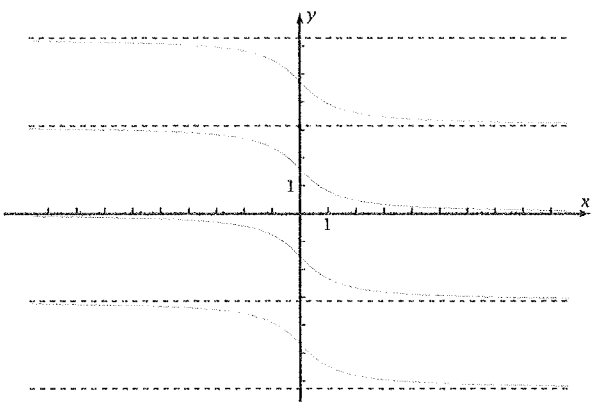
Exploration 4-6b: Principal Branches of Inverse Trigonometric Relations *continued*

Date: ______________

7. Duplicate the previous graph of $y = \arctan x$ on your grapher. Give the parametric equations you used. Check your graph with your instructor. ______

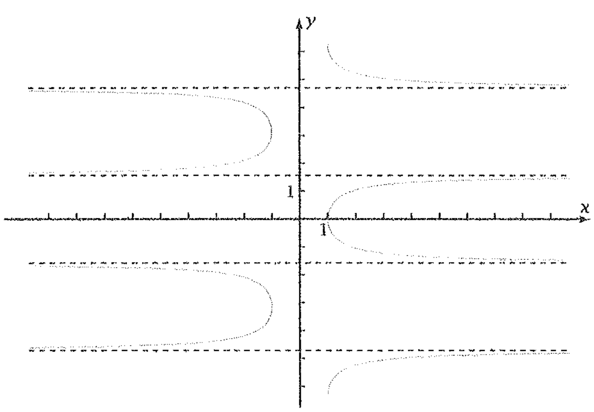
8. The graph here shows $y = \text{arccot}\ x$. How can you define the range of the function $y = \cot^{-1} x$ in such a way that the function is **continuous**? Darken this principal branch of $y = \text{arccot}\ x$.

   Range: ______________



9. This next graph shows $y = \text{arcsec}\ x$. There is no way to restrict the range to make a continuous function $y = \sec^{-1} x$ and still use all of the domain. Darken what you think would be the best choice for the principal branch. Write the range.

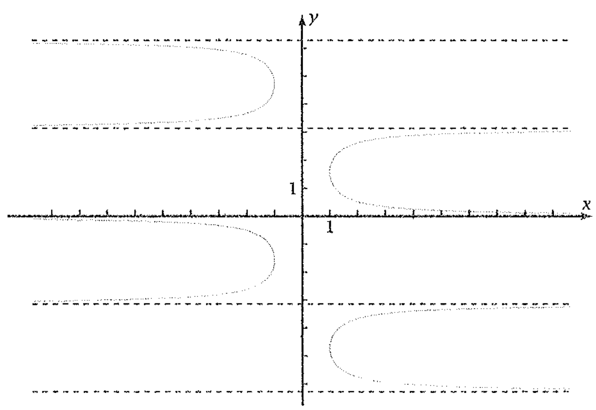
   Range: ______________



10. Look in the text to find out if the principal branch you chose in Problem 9 is the commonly accepted one. ______

11. This next graph shows $y = \text{arccsc}\ x$. Shade what you think the principal branch is. Write the range of the function you shaded.

    Range: ______________



12. Does your answer to Problem 11 agree with the range listed in the text?

13. Look up in the text the five criteria for picking the ranges of the principal branches of the six inverse circular functions. Write the criteria here.

14. What did you learn as a result of doing this Exploration that you did not know before?