

# FINAL REVIEW SOLUTIONS.

17. Dividing by 13 and 25 before taking logs gives

$$\begin{aligned}
 13e^{0.081t} &= 25e^{0.032t} \\
 \frac{e^{0.081t}}{e^{0.032t}} &= \frac{25}{13} \\
 e^{0.081t-0.032t} &= \frac{25}{13} \\
 \ln e^{0.049t} &= \ln\left(\frac{25}{13}\right) \\
 0.049t &= \ln\left(\frac{25}{13}\right) \\
 t &= \frac{1}{0.049} \ln\left(\frac{25}{13}\right) \approx 13.345.
 \end{aligned}$$

18. This equation cannot be solved analytically. Graphing  $y = 87e^{0.066t}$  and  $y = 3t + 7$  it is clear that these graphs will not intersect, which means  $87e^{0.66t} = 3t + 7$  has no solution. The concavity of the graphs ensures that they will not intersect beyond the portions of the graphs shown in Figure 4.31.

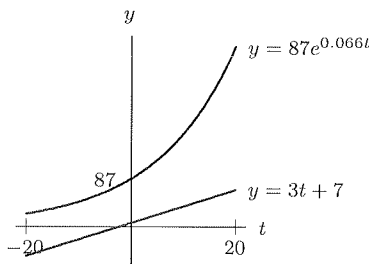


Figure 4.31

19.  $\log(100^{x+1}) = \log((10^2)^{x+1}) = 2(x+1)$ .

20.  $\ln(e \cdot e^{2+M}) = \ln(e^{3+M}) = 3 + M$ .

21. Using the fact that  $A^{-1} = 1/A$  and the log rules:

$$\begin{aligned}
 \ln(A+B) - \ln(A^{-1} + B^{-1}) &= \ln(A+B) - \ln\left(\frac{1}{A} + \frac{1}{B}\right) \\
 &= \ln(A+B) - \ln\frac{A+B}{AB} \\
 &= \ln\left((A+B) \cdot \frac{AB}{A+B}\right) \\
 &= \ln(AB).
 \end{aligned}$$

## Problems

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22. (a) After 5 years,  $B = 5000(1.12)^5 = \$8,811.71$ . After 10 years,  $B = 5000(1.12)^{10} = \$15,529.24$ .  
 (b) We need to find the value of  $t$  such that  $5000(1.12)^t = 10,000$ :

$$\begin{aligned}
 5000(1.12)^t &= 10,000 \\
 1.12^t &= 2 \quad \text{dividing by 5000}
 \end{aligned}$$

$$\begin{aligned}\log(1.12^t) &= \log 2 && \text{taking logs} \\ t \cdot \log 1.12 &= \log 2 && \text{using a log property} \\ t &= \frac{\log 2}{\log 1.12} && \text{dividing} \\ &= 6.12.\end{aligned}$$

This means it will take just over 6 years for the balance to reach \$10,000. We can use a similar approach to find out how long it takes the balance to reach \$20,000:

$$\begin{aligned}5000(1.12)^t &= 20,000 \\ 1.12^t &= 4 && \text{dividing by 5000} \\ \log(1.12^t) &= \log 4 && \text{taking logs} \\ t \cdot \log 1.12 &= \log 4 && \text{using a log property} \\ t &= \frac{\log 4}{\log 1.12} && \text{dividing} \\ &= 12.2.\end{aligned}$$

This means it will take just over 12 years for the balance to reach \$20,000.

23. We need to find the value of  $t$  for which

$$\begin{aligned}P_1 &= P_2 \\ 51(1.031)^t &= 63(1.052)^t \\ \frac{(1.031)^t}{(1.052)^t} &= \frac{63}{51} \\ \left(\frac{1.031}{1.052}\right)^t &= \frac{63}{51} \\ \log\left(\frac{1.031}{1.052}\right)^t &= \log\frac{63}{51} \\ t \log\left(\frac{1.031}{1.052}\right) &= \log\frac{63}{51} \\ t &= \frac{\log\frac{63}{51}}{\log\left(\frac{1.031}{1.052}\right)} = -10.480.\end{aligned}$$

So, the populations are the same 10.5 years before 1980, in the middle of 1969.

24. (a) If  $t$  represents the number of years since 1999, let  $W(t)$  = population of Erehwon at time  $t$ , in millions of people, and let  $C(t)$  = population of Ecalpon at time  $t$ , in millions of people. Since the population of both Erehwon and Ecalpon are increasing by a constant percent, we know that they are both exponential functions. In Erehwon, the growth factor is 1.029. Since its population in 1999 (when  $t = 0$ ) is 50 million people, we know that

$$W(t) = 50(1.029)^t.$$

In Ecalpon, the growth factor is 1.032, and the population starts at 45 million, so

$$C(t) = 45(1.032)^t.$$

- (b) The two countries will have the same population when  $W(t) = C(t)$ . We therefore need to solve:

$$\begin{aligned}50(1.029)^t &= 45(1.032)^t \\ \frac{1.032^t}{1.029^t} &= \left(\frac{1.032}{1.029}\right)^t = \frac{50}{45} = \frac{10}{9} \\ \log\left(\frac{1.032}{1.029}\right)^t &= \log\left(\frac{10}{9}\right)\end{aligned}$$

$$t \log \left( \frac{1.032}{1.029} \right) = \log \left( \frac{10}{9} \right)$$

$$t = \frac{\log \left( \frac{10}{9} \right)}{\log \left( \frac{1.032}{1.029} \right)} \approx 36.191$$

So the populations are equal after about 36.191 years.

- (c) The population of Ecalpon is double the population of Erehwon when

$$C(t) = 2W(t)$$

that is, when

$$45(1.032)^t = 2 \cdot 50(1.029)^t.$$

We use logs to solve the equation.

$$45(1.032)^t = 100(1.029)^t$$

$$\frac{(1.032)^t}{(1.029)^t} = \frac{100}{45} = \frac{20}{9}$$

$$\left( \frac{1.032}{1.029} \right)^t = \frac{20}{9}$$

$$\log \left( \frac{1.032}{1.029} \right)^t = \log \left( \frac{20}{9} \right)$$

$$t \log \left( \frac{1.032}{1.029} \right) = \log \left( \frac{20}{9} \right)$$

$$t = \frac{\log \left( \frac{20}{9} \right)}{\log \left( \frac{1.032}{1.029} \right)} \approx 274.287 \text{ years.}$$

So it will take about 274 years for the population of Ecalpon to be twice that of Erehwon.

25. (a) Let  $t$  be the number of years in a man's age above 30 (i.e. let  $t = \text{the man's age} - 30$ ) and let  $M_0$  denote his bone mass at age 30. If he is losing 2% per year, then 98% remains after each year, and thus we can say that  $M(t) = M_0(0.98)^t$ , where  $M(t)$  represents the man's bone mass  $t$  years after age 30. But we want a formula describing bone mass in terms of  $a$ , his age. Since  $t$  is number of years in his age over 30,  $t = a - 30$ . So, we can substitute  $a - 30$  for  $t$  in our formula to find an expression in terms of  $a$ :

$$M(a) = M_0(0.98)^{(a-30)}.$$

- (b) We want to know for what value of  $a$

$$M(a) = \frac{1}{2}M_0$$

Therefore, we will solve  $M_0(0.98)^{(a-30)} = \frac{1}{2}M_0$

$$(0.98)^{(a-30)} = \frac{1}{2}$$

$$\log \left( (0.98)^{(a-30)} \right) = \log \frac{1}{2} = \log 0.5$$

$$(a-30) \log(0.98) = \log 0.5$$

$$a-30 = \frac{\log 0.5}{\log 0.98}$$

$$a = 30 + \frac{\log 0.5}{\log(0.98)} \approx 64.3$$

The average man will have lost half his bone mass at approximately 64.3 years of age.

4.R

26. (a) The number of bacteria present after 1/2 hour is

$$N = 1000e^{0.69(1/2)} \approx 1412.$$

If you notice that  $0.69 \approx \ln 2$ , you could also say

$$N = 1000e^{0.69/2} \approx 1000e^{\frac{1}{2} \ln 2} = 1000e^{\ln 2^{1/2}} = 1000e^{\ln \sqrt{2}} = 1000\sqrt{2} \approx 1412.$$

- (b) We solve for  $t$  in the equation

$$\begin{aligned} 1,000,000 &= 1000e^{0.69t} \\ e^{0.69t} &= 1000 \\ 0.69t &= \ln 1000 \\ t &= \left( \frac{\ln 1000}{0.69} \right) \approx 10.011 \text{ hours.} \end{aligned}$$

- (c) The doubling time is the time  $t$  such that  $N = 2000$ , so

$$\begin{aligned} 2000 &= 1000e^{0.69t} \\ e^{0.69t} &= 2 \\ 0.69t &= \ln 2 \\ t &= \left( \frac{\ln 2}{0.69} \right) \approx 1.005 \text{ hours.} \end{aligned}$$

If you notice that  $0.69 \approx \ln 2$ , you see why the half-life turns out to be 1 hour:

$$\begin{aligned} e^{0.69t} &= 2 \\ e^{t \ln 2} &\approx 2 \\ e^{\ln 2^t} &\approx 2 \\ 2^t &\approx 2 \\ t &\approx 1 \end{aligned}$$

27. (a) If  $Q(t) = Q_0 b^t$  describes the number of gallons left in the tank after  $t$  hours, then  $Q_0$ , the amount we started with, is 250, and  $b$ , the percent left in the tank after 1 hour, is 96%. Thus  $Q(t) = 250(0.96)^t$ . After 10 hours, there are  $Q(10) = 250(0.96)^{10} \approx 166.208$  gallons left in the tank. This  $\frac{166.208}{250} = 0.66483 = 66.483\%$  of what had initially been in the tank. Therefore approximately 33.517% has leaked out. It is less than 40% because the loss is 4% of 250 only during the first hour; for each hour after that it is 4% of whatever quantity is left.
- (b) Since  $Q_0 = 250$ ,  $Q(t) = 250e^{kt}$ . But we can also define  $Q(t) = 250(0.96)^t$ , so

$$\begin{aligned} 250e^{kt} &= 250(0.96)^t \\ e^{kt} &= 0.96^t \\ e^k &= 0.96 \\ \ln e^k &= \ln 0.96 \\ k \ln e &= \ln 0.96 \\ k &= \ln 0.96 \approx -0.04082. \end{aligned}$$

Since  $k$  is negative, we know that the value of  $Q(t)$  is decreasing by 4.082% per hour. Therefore,  $k$  is the continuous hourly decay rate.

28. (a) The population has increased by  $34,000 - 30,000 = 4,000$  people in that time period, so its total percent increase is  $\frac{4,000}{30,000} = 0.13333 = 13.333\%$ .

22. This graph of the function  $y = \cos(5t + \pi/4)$  is the graph of  $y = \cos 5t$  shifted horizontally to the left. (In general, the graph of  $f(x + k)$  is the graph of  $f(x)$  shifted left if  $k$  is positive.) Since

$$5t + \frac{\pi}{4} = 5 \left( t + \frac{\pi}{20} \right),$$

the graph shifts left a distance of  $\pi/20$  units. The phase shift is  $\pi/4$ . Since  $\pi/4$  is one-eighth of the period of  $\cos t$ , the graph of this function is  $y = \cos 5t$  shifted left by one-eighth of its period.

23. From Figure 6.46, we see the amplitude  $A$  of this function is 20. We see 4 cycles in 3 seconds so the period is  $\frac{3}{4}$ .

The amplitude tells us the maximum amount by which the blood pressure can vary from its average value of 100 mm Hg. The period of  $\frac{3}{4}$  seconds tells us the duration of one cycle of blood pressure change.

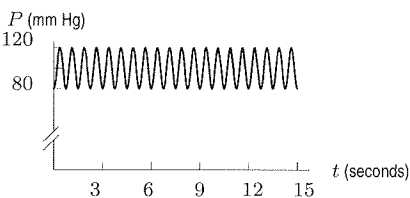


Figure 6.46

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6.5

24.  $f(t) = 17.5 + 17.5 \sin\left(\frac{2\pi}{5}t\right)$

25.  $f(t) = 14 + 10 \sin\left(\pi t + \frac{\pi}{2}\right)$

6.5

26. The amplitude and midline are 20 and the period is 5. The graph is a sine curve shifted half a period to the right (or left), so the phase shift is  $-\pi$ . Thus

$$h = 20 + 20 \sin\left(\frac{2\pi}{5}t - \pi\right).$$

27.  $f(t) = 20 + 15 \sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right)$

28. From Figure 6.5, we see that the midline and amplitude are both 225 and the period is 30. Since the graph looks like a cosine reflected about the midline, we take  $k = 225$  and  $A = -225$ . In addition

$$\frac{2\pi}{B} = 30 \quad \text{so} \quad B = \frac{\pi}{15}.$$

The cosine function does not need to be shifted horizontally, so

$$f(t) = -225 \cos\left(\frac{\pi}{15}t\right) + 225.$$

29. (a) The ferris wheel makes one full revolution in 30 minutes. Since one revolution is  $360^\circ$ , the wheel turns

$$\frac{360}{30} = 12^\circ \text{ per minute.}$$

- (b) The angle representing your position, measured from the 6 o'clock position, is  $12t^\circ$ . However, the angle shown in Figure 6.47 is measured from the 3 o'clock position, so

$$\theta = (12t - 90)^\circ.$$

- (b) The midline,  $P = 1750$ , is the average population value over one year. The period is 12 months (or 1 year), which means the cycle repeats annually. The amplitude is the amount that the population varies above and below the average annual population.
- (c) Figure 6.49 is a graph of  $f(t) = -450 \cos(\frac{\pi}{6}t) + 1750$  and  $P = 1500$ .

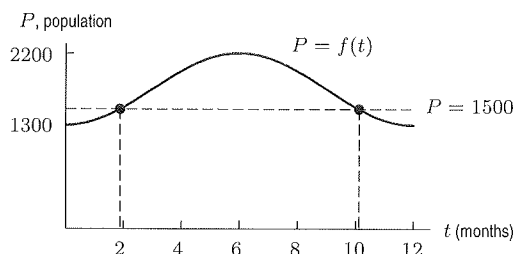


Figure 6.49

From a graph we get approximations of  $t_1 \approx 1.9$  and  $t_2 \approx 10.1$ . This means that the population is 1500 sometime in late February and again sometime in early November.

- 6-5
32. The data given describe a trigonometric function shifted vertically because all the  $g(x)$  values are greater than 1. Since the maximum is approximately 3 and the minimum approximately 1, the midline value is 2. We choose the sine function over the cosine function because the data tell us that at  $x = 0$  the function takes on its midline value, and then increases. Thus our function will be of the form

$$g(x) = A \sin(Bx) + k.$$

We know that  $A$  represents the amplitude,  $k$  represents the vertical shift, and the period is  $2\pi/B$ .

We've already noted the midline value is  $k \approx 2$ . This means  $A = \max - k = 1$ . We also note that the function completes a full cycle after 1 unit. Thus

$$1 = \frac{2\pi}{B}$$

so

$$B = 2\pi.$$

Thus

$$g(x) = \sin(2\pi x) + 2.$$

33. This function has an amplitude of 3 and a period 1, and resembles a sine graph. Thus  $y = 3f(x)$ .
34. This function has an amplitude of 2 and a period of 3, and resembles vertically reflected cosine graph. Thus  $y = -2g(x/3)$ .
35. This function has an amplitude of 1 and a period of 0.5, and resembles an inverted sine graph. Thus  $y = -f(2x)$ .
36. This function has an amplitude of 2 and a period of 1 and a midline of  $y = -3$ , and resembles a cosine graph. Thus  $y = 2g(x) - 3$ .
- 37.

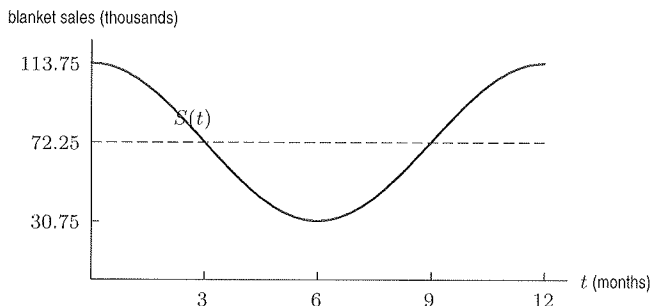


Figure 6.50

14. We start with  $f(f^{-1}(x)) = x$  and substitute  $y = f^{-1}(x)$ . We have

$$\begin{aligned} f(y) &= x \\ \cos \sqrt{y} &= x \\ \sqrt{y} &= \arccos x \\ y &= (\arccos x)^2. \end{aligned}$$

Thus

$$f^{-1}(x) = (\arccos x)^2.$$

15. We start with  $g(g^{-1}(x)) = x$  and substitute  $y = g^{-1}(x)$ . We have

$$\begin{aligned} g(y) &= x \\ 2^{\sin y} &= x \\ \ln(2^{\sin y}) &= \ln x \\ \ln(2^{\sin y}) &= (\sin y) \ln 2 = \ln x \\ \sin y &= \frac{\ln x}{\ln 2} \\ y &= \arcsin\left(\frac{\ln x}{\ln 2}\right). \end{aligned}$$

Thus

$$g^{-1}(x) = \arcsin\left(\frac{\ln x}{\ln 2}\right).$$

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8.R

### Problems

16. (a) To evaluate  $g(f(23))$ , we first need to evaluate  $f(23)$ . According to Table 8.28 in the problem,  $f(23) = -5$ . Thus,  $g(f(23)) = g(-5)$ . Using Table 8.29 in the problem, we find that  $g(-5) = 23$ . Therefore,

$$g(f(23)) = 23.$$

Since  $f(23)$  means "the temperature Celsius that corresponds to 23 degrees Fahrenheit,  $g(f(23))$  is equivalent to the temperature in degrees Fahrenheit corresponding to the temperature in degrees Celsius that equals 23° Fahrenheit. In evaluating  $g(f(23))$ , we start with 23 and come full circle back to 23.

- (b) To evaluate  $f(g(5))$ , we first note that since  $g(5) = 41$ ,  $f(g(5)) = f(41) = 5$ . We started at a particular temperature, this time in degrees Celsius. The function  $g$  converted this temperature to the equivalent temperature in degrees Fahrenheit. Then, the function  $f$  converted this Fahrenheit temperature back to the equivalent Celsius temperature, which was, not surprisingly, where we started in the first place.
17. Using  $f(x) = \frac{1}{x+1}$ , we obtain

$$\begin{aligned} f\left(\frac{1}{x}\right) + \frac{1}{f(x)} &= \frac{1}{\frac{1}{x} + 1} + \frac{1}{\frac{1}{x+1}} \\ &= \frac{1}{\frac{1+x}{x}} + x + 1 \\ &= \frac{x}{1+x} + x + 1 \end{aligned}$$

18.  $g(x) = x^2$  and  $h(x) = x + 3$

19.  $g(x) = \sqrt{x}$  and  $h(x) = 1 + \sqrt{x}$

20.  $g(x) = x^2 + x$  and  $h(x) = 3x$

21.  $g(x) = \frac{1}{x^2}$  and  $h(x) = x + 4$

22. Since  $f(x) = x - 3$ , we have  $f(h(x)) = h(x) - 3$ , which means  $h(x) - 3 = \sqrt{x}$ . Thus,  $h(x) = \sqrt{x} + 3$ .

23. Since  $g(x) = 2x + 5$ , we have  $g(h(x)) = 2h(x) + 5 = 6x - 7$ . Thus,  $2h(x) = 6x - 12$ , which gives  $h(x) = 3x - 6$ .

24. Since  $h(g(x)) = h(2x + 5)$ , we have

$$h(2x + 5) = \frac{2x + 5}{1 + \sqrt{2x + 5}},$$

which means

$$h(x) = \frac{x}{1 + \sqrt{x}}.$$

25. Since  $h(g(x)) = h(2x + 5)$ , we have

$$\begin{aligned} h(2x + 5) &= \frac{2x + 6}{1 + \sqrt{2x + 4}} \\ &= \frac{2x + 5 + 1}{1 + \sqrt{2x + 5} - 1} \end{aligned}$$

which means

$$h(x) = \frac{x + 1}{1 + \sqrt{x - 1}}$$

26. See Figure 8.32.

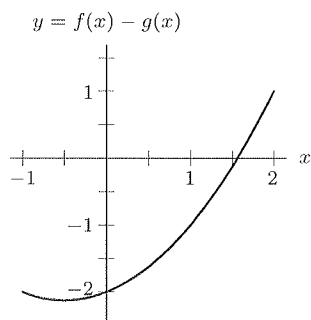


Figure 8.32

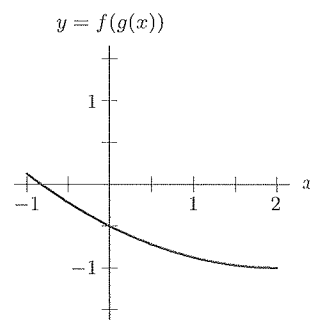


Figure 8.33

27. See Figure 8.33.

34. (a) See Figure 8.40.

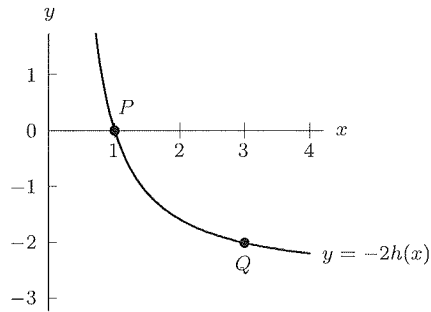


Figure 8.40:  $y = -2h(x)$

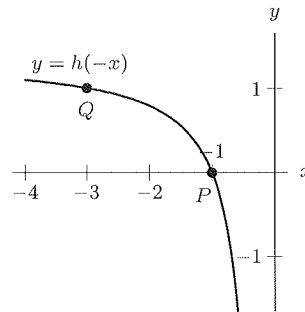


Figure 8.41:  $y = h(-x)$

(b) See Figure 8.41.  
 (c) See Figure 8.42.

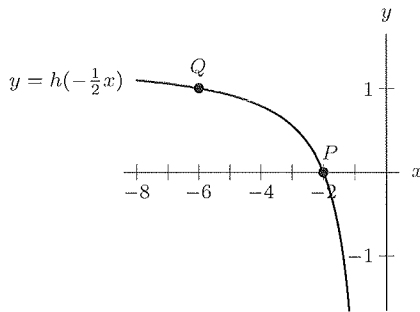


Figure 8.42:  $y = h(-\frac{1}{2}x)$

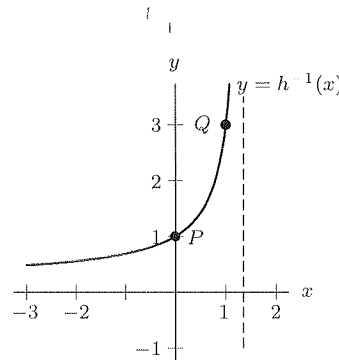


Figure 8.43:  $y = h^{-1}(x)$

(d) See Figure 8.43.

35. (a) Graph II, because the graph of  $y = -f(x)$  is the reflection of  $f(x)$  across the  $x$ -axis.  
 (b) Graph I, because the graph of  $y = f(-x)$  reflects the graph across the  $y$ -axis.  
 (c) None. This graph would look like Graph I but with a  $y$ -intercept of  $-1$ .  
 (d) None. The graph of  $f^{-1}(x)$  would be a reflection of the graph of  $f(x)$  about the line  $y = x$ .  
 (e) Graph IV. This is the graph of  $f(x)$  reflected about the line  $y = x$  and then reflected about the  $x$ -axis.  
 (f) None. Adding 1 to the argument, we shift the graph of  $f(x)$  to the left by 1.  
 (g) None. This is the graph of  $f(x)$  reflected about the  $x$ -axis and raised by 2.

8-R

36. (a)  $f(g(a)) = f(a) = a$   
 (b)  $g(f(c)) = g(c) = b$   
 (c)  $f^{-1}(b) - g^{-1}(b) = 0 - c = -c$   
 (d)  $0 < x \leq a$

37. Solving for  $r$  gives

$$r = \log\left(\frac{I}{I_0}\right)$$

$$r = \log\left(\frac{I}{I_0}\right)$$

$$\frac{I}{I_0} = 10^r$$

$$I = I_0 10^r.$$

The inverse function  $f^{-1}(r) = I_0 10^r$  gives the intensity of an earthquake with Richter rating  $r$ .

26. We know that  $v$  and  $u$  should not have the exact same graph. On the window  $-10 \leq x \leq 10$  by  $-10 \leq y \leq 10$  the graphs appear to be identical, but we know the functions are not the same, because  $u$  has a repeated zero at  $x = 0$ . The function  $v$  has a zero at  $x = 0$  and repeated zeros at  $x = 0.01$ . In order to see the differences, we must choose a much smaller window. One choice which clearly shows the differences is  $-0.01 \leq x \leq 0.01$  by  $-0.0000001 \leq y \leq 0.0000001$ .

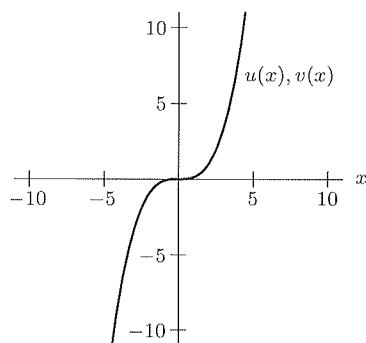


Figure 9.21

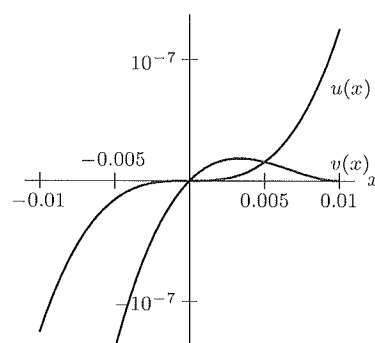


Figure 9.22

27. Since all the three points fall on a horizontal line, the constant function  $f(x) = 1$  (degree zero) is the only polynomial of degree  $\leq 2$  to satisfy the given conditions.

28. To pass through the given points, the polynomial must be of at least degree 2. Thus, let  $f$  be of the form

$$f(x) = ax^2 + bx + c.$$

Then using  $f(0) = 0$  gives

$$a(0)^2 + b(0) + c = 0,$$

so  $c = 0$ . Then, with  $f(2) = 0$ , we have

$$\begin{aligned} a(2)^2 + b(2) + 0 &= 0 \\ 4a + 2b &= 0 \\ \text{so } b &= -2a. \end{aligned}$$

Using  $f(3) = 3$  and  $b = -2a$  gives

$$a(3)^2 + (-2a)(3) + 0 = 3$$

so

$$\begin{aligned} 9a - 6a &= 3 \\ 3a &= 3 \\ a &= 1. \end{aligned}$$

Thus,  $b = -2a$  gives  $b = -2$ . The unique polynomial of degree  $\leq 2$  which satisfies the given conditions is  $f(x) = x^2 - 2x$ .

29. Clearly  $f(x) = x$  works. However, the solution is not unique. If  $f$  is of the form  $f(x) = ax^2 + bx + c$ , then  $f(0) = 0$  gives  $c = 0$ , and  $f(1) = 1$  gives

$$a(1)^2 + b(1) + 0 = 1,$$

so

$$a + b = 1,$$

or

$$b = 1 - a.$$

Since these are the only conditions which must be satisfied, any polynomial of the form

$$f(x) = ax^2 + (1 - a)x$$

will work. If  $a = 0$ , we get  $f(x) = x$ .

p. 395

9.3

33. (a) We could think of  $f(x) = (x - 2)^3 + 4$  as  $y = x^3$  shifted right 2 units and up 4. Thus, since  $y = x^3$  is invertible,  $f$  should also be. Algebraically, we let

$$y = f(x) = (x - 2)^3 + 4.$$

Thus,

$$\begin{aligned} y - 4 &= (x - 2)^3 \\ \sqrt[3]{y - 4} &= x - 2 \\ \sqrt[3]{y - 4} + 2 &= x \end{aligned}$$

So  $f(x)$  is invertible with an inverse

$$f^{-1}(x) = \sqrt[3]{x - 4} + 2.$$

- (b) Since  $g$  is not so obvious, we might begin by graphing  $y = g(x)$ . Figure 9.23 shows that the function  $g(x) = x^3 - 4x^2 + 2$  does not satisfy the horizontal-line-test, so  $g$  is not invertible.

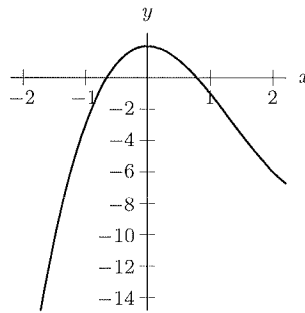


Figure 9.23

34. Factoring  $y = x^2 + 5x + 6$  gives  $y = (x + 2)(x + 3)$ . Thus  $y = 0$  for  $x = -2$  or  $x = -3$ .
35. Note that  $y = x^4 + 6x^2 + 9 = (x^2 + 3)^2$ . This implies that  $y = 0$  if  $x^2 = -3$ , but  $x^2 = -3$  has no real solutions. Thus, there are no zeros.
- 9.3 36.  $y = 4x^2 - 1 = (2x - 1)(2x + 1)$ , which implies that  $y = 0$  for  $x = \pm \frac{1}{2}$ .
37.  $y = 4x^2 + 1 = 0$  implies that  $x^2 = -\frac{1}{4}$ , which has no solutions. There are no real zeros.
38. By using the quadratic formula, we find that  $y = 0$  if

$$x = \frac{3 \pm \sqrt{9 - 4(2)(-3)}}{4} = \frac{3 \pm \sqrt{33}}{4}.$$

39. This problem cannot be solved algebraically. Note that we cannot use the quadratic formula, as this is a 5th degree polynomial and not a 2nd degree polynomial. A graph of the function is shown in Figure 9.24 for  $-1 \leq x \leq 1$ ,  $-10 \leq y \leq 10$ . From the graph, we approximate the zero to be at  $x \approx -0.143$ .

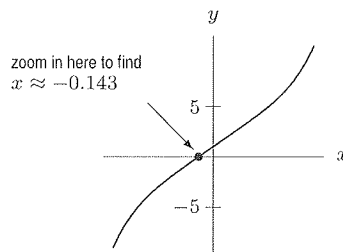


Figure 9.24

The formula for  $V$  is a difference of two third-degree polynomials. The format is not terribly convenient, so we simplify the formula by multiplying the factors for  $V_b$  and  $V_c$  and gathering like terms. Then for  $V$  we have

$$\begin{aligned} V &= (x+1)(x+3)(x+1) - x(x+2)(x-1) \\ &= (x^3 + 5x^2 + 7x + 3) - (x^3 + x^2 - 2x) \\ &= 4x^2 + 9x + 3. \end{aligned}$$

The formula  $V(x) = 4x^2 + 9x + 3$  gives the necessary information for *appropriate values* of  $x$ . Note that the quadratic function  $y = 4x^2 + 9x + 3$  is defined for all values of  $x$ . However, since  $x$  represents the length of a box and  $(x-1)$  is the depth of the box, the formula only makes sense as a model for  $x > 1$ . In this case, an understanding of the component polynomials representing  $V_b$  and  $V_c$  is necessary in order to determine the logical domain for  $V(x)$ .

9.3 (42.) We express the volume as a function of the length,  $x$ , of the square's side that is cut out in Figure 9.26.

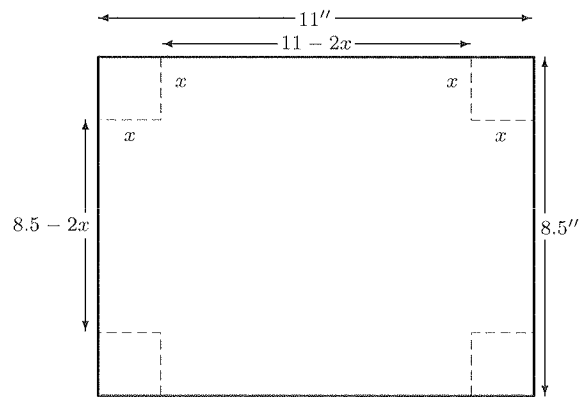


Figure 9.26

Since the sides of the base are  $(11 - 2x)$  and  $(8.5 - 2x)$  inches and the depth is  $x$  inches, the volume,  $V(x)$ , is given by

$$V(x) = x(11 - 2x)(8.5 - 2x).$$

The graph of  $V(x)$  in Figure 9.27 suggests that the maximum volume occurs when  $x \approx 1.585$  inches. (A good viewing window is  $0 \leq x \leq 5$  and  $0 \leq y \leq 70$ .) So one side is  $x = 1.585$ , and therefore the others are  $11 - 2(1.585) = 7.83$  and  $8.5 - 2(1.585) = 5.33$ .

The dimensions of the box are 7.83 by 5.33 by 1.585 inches.

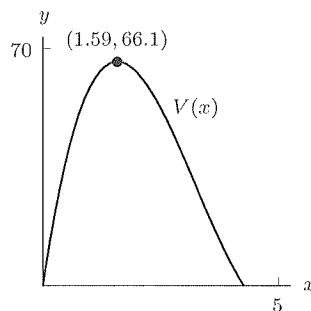


Figure 9.27

16. (a) The displacement from  $P$  to  $Q$  is given by

$$\overrightarrow{PQ} = (4\vec{i} + 6\vec{j}) - (\vec{i} + 2\vec{j}) = 3\vec{i} + 4\vec{j}.$$

Since

$$\|\overrightarrow{PQ}\| = \sqrt{3^2 + 4^2} = 5,$$

a unit vector  $\vec{u}$  in the direction of  $\overrightarrow{PQ}$  is given by

$$\vec{u} = \frac{1}{5}\overrightarrow{PQ} = \frac{1}{5}(3\vec{i} + 4\vec{j}) = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}.$$

- (b) A vector of length 10 pointing in the same direction is given by

$$10\vec{u} = 10\left(\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}\right) = 6\vec{i} + 8\vec{j}.$$

17. We need to calculate the length of each vector.

$$\|21\vec{i} + 35\vec{j}\| = \sqrt{21^2 + 35^2} = \sqrt{1666} \approx 40.8,$$

$$\|40\vec{i}\| = \sqrt{40^2} = 40.$$

So the first car is faster.

18. Since both vehicles reach the crossroad in exactly one hour, at the present the truck is at  $O$  in Figure 10.20; the police car is at  $P$  and the crossroads is at  $C$ . If  $\vec{r}$  is the vector representing the displacement of the truck with respect to the police car:

$$\vec{r} = -40\vec{i} - 30\vec{j}.$$

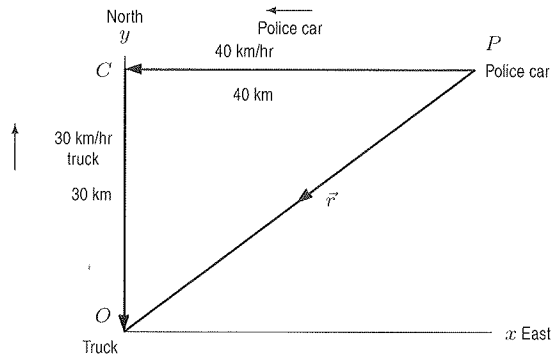


Figure 10.20

19. (a) If the car is going east, it is going solely in the positive  $x$ -direction, so its velocity vector is  $50\vec{i}$ . The  $\vec{j}$ -component is  $\vec{0}$ .  
 (b) If the car is going south, it is going solely in the negative  $y$ -direction, so its velocity vector is  $-50\vec{j}$ . The  $\vec{i}$ -component is  $\vec{0}$ .  
 (c) If the car is going southeast, the angle between the  $x$ -axis and the velocity vector is  $-45^\circ$ . Therefore

$$\begin{aligned} \text{Velocity vector} &= 50 \cos(-45^\circ)\vec{i} + 50 \sin(-45^\circ)\vec{j} \\ &= (25\sqrt{2})\vec{i} - (25\sqrt{2})\vec{j}. \end{aligned}$$

- (d) If the car is going northwest, the velocity vector is at a  $45^\circ$  angle to the  $y$ -axis, which is  $135^\circ$  from the  $x$ -axis. Therefore:

$$\text{Velocity vector} = 50(\cos 135^\circ)\vec{i} + 50(\sin 135^\circ)\vec{j} = -(25\sqrt{2})\vec{i} + (25\sqrt{2})\vec{j}.$$

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10.2

- 10.2 23. We get displacement by subtracting the coordinates of the origin  $(0, 0, 0)$  from the coordinates of the cat  $(1, 4, 0)$ , giving

$$\text{Displacement} = (1 - 0)\vec{i} + (4 - 0)\vec{j} + (0 - 0)\vec{k} = \vec{i} + 4\vec{j}.$$

24. We get displacement by subtracting the coordinates of the bottom of the tree,  $(2, 4, 0)$ , from the coordinates of the squirrel,  $(2, 4, 1)$ , giving:

$$\text{Displacement} = (2 - 2)\vec{i} + (4 - 4)\vec{j} + (1 - 0)\vec{k} = \vec{k}.$$

25.

$$\begin{aligned} \text{Displacement} &= \text{Cat's coordinates} - \text{Bottom of the tree's coordinates} \\ &= (1 - 2)\vec{i} + (4 - 4)\vec{j} + (0 - 0)\vec{k} = -\vec{i}. \end{aligned}$$

26.

$$\begin{aligned} \text{Displacement} &= \text{Squirrel's coordinates} - \text{Cat's coordinates} \\ &= (2 - 1)\vec{i} + (4 - 4)\vec{j} + (1 - 0)\vec{k} = \vec{i} + \vec{k}. \end{aligned}$$

## Solutions for Section 10.3

### Exercises

- $\vec{C} = \vec{N} + \vec{M} = (5, 6, 7, 8, 9, 10) + (1, 1, 2, 3, 5, 8) = (6, 7, 9, 11, 14, 18).$
- $\vec{A} = \vec{M} - \vec{N} = (1, 1, 2, 3, 5, 8) - (5, 6, 7, 8, 9, 10) = (-4, -5, -5, -5, -4, -2).$
- $\vec{e} = 2\vec{N} - 7\vec{M} = 2(5, 6, 7, 8, 9, 10) - 7(1, 1, 2, 3, 5, 8) = (10, 12, 14, 16, 18, 20) - (7, 7, 14, 21, 35, 56) = (3, 5, 0, -5, -17, -36).$
- $\vec{Z} = \frac{\vec{N}}{3} + \frac{\vec{M}}{2} = \frac{1}{3}(5, 6, 7, 8, 9, 10) + \frac{1}{2}(1, 1, 2, 3, 5, 8) = \left(\frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}, 3, \frac{10}{3}\right) + \left(\frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}, \frac{5}{2}, 4\right) = \left(\frac{13}{6}, \frac{5}{2}, \frac{10}{3}, \frac{25}{6}, \frac{11}{2}, \frac{22}{3}\right).$
- $\vec{K} = \frac{\vec{N}}{3} + \frac{2\vec{N}}{3} = \frac{3\vec{N}}{3} = \vec{N} = (5, 6, 7, 8, 9, 10).$
- $\vec{p} = 1.067\vec{M} + 2.361\vec{N} = 1.067(1, 1, 2, 3, 5, 8) + 2.361(5, 6, 7, 8, 9, 10) = (1.067, 1.067, 2.134, 3.201, 5.335, 8.536) + (11.805, 14.166, 16.527, 18.888, 21.249, 23.61) = (12.872, 15.233, 18.661, 22.089, 26.584, 32.146).$
- Since each population increases by 2%, we multiply  $\vec{Q}$  by 1.02 to find  $\vec{R}$ :
 
$$\vec{R} = 1.02\vec{Q} = 1.02(3.43, 1.29, 6.38, 1.26, 1.06, 0.61) = (3.499, 1.316, 6.508, 1.285, 1.081, 0.622).$$
- Since the components of  $\vec{Q}$  represent millions of people, an increase of 120,000 people will increase each component by 0.12. Therefore,
 
$$\begin{aligned} \vec{S} &= \vec{Q} + (0.12, 0.12, 0.12, 0.12, 0.12, 0.12) \\ &= (3.43, 1.29, 6.38, 1.26, 1.06, 0.61) + (0.12, 0.12, 0.12, 0.12, 0.12, 0.12) \\ &= (3.55, 1.41, 6.50, 1.38, 1.18, 0.73). \end{aligned}$$

12. (a) The force has magnitude 2 downward and the displacement has magnitude 3 downward. Therefore  $\theta = 0^\circ$ , so

$$\text{Work done} = 2 \cdot 3 \cos 0^\circ = 2 \cdot 3 \cdot 1 = 6 \text{ ft}\cdot\text{lb.}$$

- (b) The force is the same, the displacement has magnitude 5 feet and the angle between the force and the displacement is  $180^\circ$ . Thus,

$$\text{Work done} = 2 \cdot 5 \cos 180^\circ = 2 \cdot 5(-1) = -10 \text{ ft}\cdot\text{lb.}$$

13. Since  $3\vec{i} + \sqrt{3}\vec{j} = \sqrt{3}(\sqrt{3}\vec{i} + \vec{j})$ , we know that  $3\vec{i} + \sqrt{3}\vec{j}$  and  $\sqrt{3}\vec{i} + \vec{j}$  are scalar multiples of one another, and therefore parallel.

Since  $(\sqrt{3}\vec{i} + \vec{j}) \cdot (\vec{i} - \sqrt{3}\vec{j}) = \sqrt{3} - \sqrt{3} = 0$ , we know that  $\sqrt{3}\vec{i} + \vec{j}$  and  $\vec{i} - \sqrt{3}\vec{j}$  are perpendicular.

Since  $3\vec{i} + \sqrt{3}\vec{j}$  and  $\sqrt{3}\vec{i} + \vec{j}$  are parallel,  $3\vec{i} + \sqrt{3}\vec{j}$  and  $\vec{i} - \sqrt{3}\vec{j}$  are perpendicular, too.

14. Using the dot product, the angle is given by

$$\cos \theta = \frac{(\vec{i} + \vec{j} + \vec{k}) \cdot (\vec{i} - \vec{j} - \vec{k})}{\|\vec{i} + \vec{j} + \vec{k}\| \|\vec{i} - \vec{j} - \vec{k}\|} = \frac{1 \cdot 1 + 1(-1) + 1(-1)}{\sqrt{1^2 + 1^2 + 1^2} \sqrt{1^2 + (-1)^2 + (-1)^2}} = -\frac{1}{3}.$$

So,  $\theta = \arccos(-\frac{1}{3}) \approx 1.911$  radians, or  $\approx 109.471^\circ$ .

15. In general,  $\vec{u}$  and  $\vec{v}$  are perpendicular when  $\vec{u} \cdot \vec{v} = 0$ .

In this case,  $\vec{u} \cdot \vec{v} = (t\vec{i} - \vec{j} + \vec{k}) \cdot (t\vec{i} + t\vec{j} - 2\vec{k}) = t^2 - t - 2$ .

This is zero when  $t^2 - t - 2 = 0$ , i.e. when  $(t-2)(t+1) = 0$ , so  $t = 2$  or  $-1$ .

In general,  $\vec{u}$  and  $\vec{v}$  are parallel if and only if  $\vec{v} = \alpha\vec{u}$  for some real number  $\alpha$ .

Thus we need  $\alpha t\vec{i} - \alpha\vec{j} + \alpha\vec{k} = t\vec{i} + t\vec{j} - 2\vec{k}$ , so we need  $\alpha t = t$ , and  $-\alpha = t$ , and  $\alpha = -2$ . But if  $\alpha = -2$ , we can't have  $\alpha t = t$  unless  $t = 0$ , and if  $t = 0$ , we can't have  $-\alpha = t$ , so there are no values of  $t$  for which  $\vec{u}$  and  $\vec{v}$  are parallel.

16. The maximum value of  $\|\vec{a} + \vec{b}\|$  is  $7 + 4 = 11$ , which occurs when  $\vec{a}$  and  $\vec{b}$  are in the same direction. The minimum value of  $\|\vec{a} + \vec{b}\|$  is  $7 - 4 = 3$ , which occurs when they are in opposite directions.

The maximum value of  $\|\vec{a} - \vec{b}\|$  is  $7 + 4 = 11$ , which occurs when  $\vec{a}$  and  $\vec{b}$  are in opposite directions. The minimum value of  $\|\vec{a} - \vec{b}\|$  is  $7 - 4 = 3$ , which occurs when  $\vec{a}$  and  $\vec{b}$  are in the same direction.

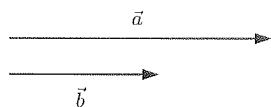


Figure 10.32: Vectors  $\vec{a}$  and  $\vec{b}$  in same direction: Max  $\|\vec{a} + \vec{b}\|$ , min  $\|\vec{a} - \vec{b}\|$



Figure 10.33: Vectors  $\vec{a}$  and  $\vec{b}$  in opposite direction: max  $\|\vec{a} - \vec{b}\|$ , min  $\|\vec{a} + \vec{b}\|$

17. We need to find the speed of the wind in the direction of the track. Looking at Figure 10.34, we see that we want the component of  $\vec{w}$  in the direction of  $\vec{v}$ . We calculate

$$\begin{aligned} \|\vec{w}_{\text{parallel to } \vec{v}}\| &= \|\vec{w}\| \cos \theta = \frac{\vec{w} \cdot \vec{v}}{\|\vec{v}\|} = \frac{(5\vec{i} + \vec{j}) \cdot (2\vec{i} + 6\vec{j})}{\|2\vec{i} + 6\vec{j}\|} \\ &= \frac{16}{\sqrt{40}} \approx 2.53 < 5. \end{aligned}$$

Therefore, the race results will not be disqualified.

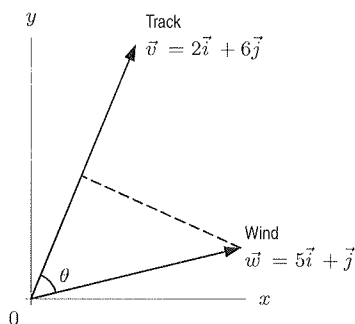


Figure 10.34

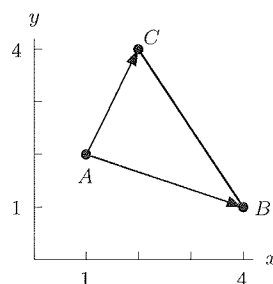


Figure 10.35

- 10.4 18. It is clear from the Figure 10.35 that only angle  $\angle CAB$  could possibly be a right angle. Subtraction of  $x, y$  values for the points gives  $\vec{AB} = 3\vec{i} - \vec{j}$  and  $\vec{AC} = 1\vec{i} + 2\vec{j}$ . Taking the dot product yields  $\vec{AB} \cdot \vec{AC} = (3)(1) + (-1)(2) = 1$ . Since this is non-zero, the angle can not be a right angle.

19. We have:

$$\begin{aligned}\vec{u} \cdot (\vec{v} + \vec{w}) &= (u_1, u_2) \cdot (v_1 + w_1, v_2 + w_2) \\ &= u_1(v_1 + w_1) + u_2(v_2 + w_2) = u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2 \\ &= (u_1v_1 + u_2v_2) + (u_1w_1 + u_2w_2) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}.\end{aligned}$$

- 10.4 20. We want to show that  $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}$  and  $\vec{c}$  are perpendicular. We do this by taking their dot product:

$$((\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}) \cdot \vec{c} = (\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{c}) = 0.$$

Since the dot product is 0, the vectors  $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}$  and  $\vec{c}$  are perpendicular.

21. By the distributive law,

$$\begin{aligned}\vec{u} \cdot \vec{v} &= (u_1\vec{i} + u_2\vec{j}) \cdot (v_1\vec{i} + v_2\vec{j}) \\ &= u_1v_1\vec{i} \cdot \vec{i} + u_1v_2\vec{i} \cdot \vec{j} + u_2v_1\vec{j} \cdot \vec{i} + u_2v_2\vec{j} \cdot \vec{j}.\end{aligned}$$

Since  $\|\vec{i}\| = \|\vec{j}\| = 1$  and the angle between  $\vec{i}$  and  $\vec{j}$  is  $90^\circ$ ,

$$\vec{i} \cdot \vec{i} = 1, \quad \vec{j} \cdot \vec{j} = 1, \quad \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{i} = 0.$$

Thus,

$$\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2.$$

22. (a) We have the price vector  $\vec{a} = (1.5, 1, 2)$ . Let the consumption vector  $\vec{c} = (c_b, c_e, c_m)$ , then  $1.5c_b + c_e + 2c_m = 20$  or  $\vec{a} \cdot \vec{c} = 20$ .
- (b) Note  $\vec{a} \cdot \vec{c}$  is the cost of consuming  $\vec{c}$  groceries at Acme Store, so  $\vec{b} \cdot \vec{c}$  is the cost of consuming  $\vec{c}$  groceries at Beta Mart. Thus  $\vec{b} \cdot \vec{c} - \vec{a} \cdot \vec{c} = (\vec{b} - \vec{a}) \cdot \vec{c}$  is the difference in costs between Beta and Acme for the same  $\vec{c}$  groceries. For  $\vec{b} - \vec{a}$  to be perpendicular to  $\vec{c}$ , we must have  $(\vec{b} - \vec{a}) \cdot \vec{c} = 0$ . Since  $\vec{b} - \vec{a} = (0.10, -0.10, 0.25)$ , the vector  $\vec{b} - \vec{a}$  is perpendicular to  $\vec{c}$  if  $0.10c_b - 0.10c_e + 0.25c_m = 0$ . For example, this occurs when we consume the same amount of bread and eggs but no milk.
- (c) Since  $\vec{b} \cdot \vec{c}$  is the cost of groceries at Beta, you might think of  $(1/1.1)\vec{b} \cdot \vec{c}$  as the "freshness-adjusted" cost at Beta. Then  $(1/1.1)\vec{b} \cdot \vec{c} < \vec{a} \cdot \vec{c}$  means the "freshness-adjusted" cost is cheaper at Beta.

(e) Either graph the function and see where it reaches its maximum value or, solve algebraically. Complete the square:

$$\begin{aligned}y &= -16t^2 + 48t + 6 \\y &= -16\left(t^2 - 3t + \frac{9}{4}\right) + 6 + 36 \\&= -16\left(t - \frac{3}{2}\right)^2 + 42.\end{aligned}$$

At 1.5 seconds the ball reaches its maximum height of 42 feet above the ground.

## Solutions for Section 12.2

### Exercises

1. Dividing by 4 and rewriting the equation as

$$x^2 + y^2 = \frac{9}{4},$$

we see that the center is  $(0, 0)$ , and the radius is  $3/2$ .

2. Dividing by 3 and rewriting the equation as

$$x^2 + y^2 = \frac{10}{3},$$

we see that the center is  $(0, 0)$ , and the radius is  $\sqrt{10/3}$ .

3. Moving the  $-(x+1)^2$  term to the left and dividing by 2 gives

$$\begin{aligned}2(x+1)^2 + 2(y+3)^2 &= 32 \\(x+1)^2 + (y+3)^2 &= 16.\end{aligned}$$

Thus, the center is  $(-1, -3)$ , and the radius is 4.

4. Multiplying by  $(y-4)^2$  gives

$$\begin{aligned}4 - (x-4)^2 - (y-4)^2 &= 0 \\(x-4)^2 + (y-4)^2 &= 4.\end{aligned}$$

Thus, the center is  $(4, 4)$ , and the radius is 2.

5. We consider  $x = 4 \cos t$ ,  $y = 4 \sin t$  for  $0 \leq t \leq 2\pi$ . However, since  $y$  increases from 0 as  $t$  increases from 0, this goes counterclockwise. We can use

$$x = 4 \cos t, \quad y = -4 \sin t \quad \text{for } 0 \leq t \leq 2\pi.$$

6. We can use  $x = 3 \sin t$ ,  $y = 3 \cos t$  for  $0 \leq t \leq 2\pi$ .

7. We can use  $x = 5 \sin t$ ,  $y = -5 \cos t$  for  $0 \leq t \leq 2\pi$ .

8. We consider  $x = -7 \cos t$ ,  $y = 7 \sin t$  for  $0 \leq t \leq 2\pi$ . However, as  $t$  increases from 0, the value of  $y$  increases, so this starts at the correct point but goes clockwise. We can use

$$x = -7 \cos t, \quad y = -7 \sin t, \quad \text{for } 0 \leq t \leq 2\pi.$$

9. We can use  $x = 3 + 5 \cos t$ ,  $y = 4 + 5 \sin t$  for  $0 \leq t \leq 2\pi$ .

10. We can use  $x = 3 + 4 \sin t$ ,  $y = 1 + 4 \cos t$  for  $0 \leq t \leq 2\pi$ .

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12.2

2. (a) The center is at the origin. The diameter in the  $x$ -direction is 4 and the diameter in the  $y$ -direction is  $2\sqrt{5}$ .  
 (b) The equation of the ellipse is

$$\frac{x^2}{2^2} + \frac{y^2}{(\sqrt{5})^2} = 1 \quad \text{or} \quad \frac{x^2}{4} + \frac{y^2}{5} = 1.$$

3. (a) The center is at  $(5, 3)$ . The diameter in the  $x$ -direction is 8 and the diameter in the  $y$ -direction is 6.  
 (b) The equation of the ellipse is

$$\frac{(x-5)^2}{4^2} + \frac{(y-3)^2}{3^2} = 1 \quad \text{or} \quad \frac{(x-5)^2}{16} + \frac{(y-3)^2}{9} = 1.$$

4. (a) The center is  $(-2, -1)$ . The diameter in the  $x$ -direction is 6 and the diameter in the  $y$ -direction is 10.  
 (b) The equation of the ellipse is

$$\frac{(x+2)^2}{3^2} + \frac{(y+1)^2}{5^2} = 1 \quad \text{or} \quad \frac{(x+2)^2}{9} + \frac{(y+1)^2}{25} = 1.$$

5. We can use  $x = 12 \cos t$ ,  $y = 5 \sin t$  for  $0 \leq t \leq 2\pi$ .

6. We consider  $x = -2 \cos t$ ,  $y = 5 \sin t$ , for  $0 \leq t \leq 2\pi$ . However, in this parameterization,  $y$  increases as  $t$  increases from 0, so it traces clockwise. Thus, we take  $x = -2 \cos t$ ,  $y = -5 \sin t$ , for  $0 \leq t \leq 2\pi$ .

7. The center is  $(5, 3)$ , and  $a = 4$ ,  $b = 3$ , so we consider  $x = 5 + 4 \cos t$ ,  $y = 3 + 3 \sin t$ , for  $0 \leq t \leq 2\pi$ . However, this parameterization traces out the ellipse counterclockwise, so we use  $x = 5 + 4 \cos t$ ,  $y = 3 - 3 \sin t$ , for  $0 \leq t \leq 2\pi$ .

8. The center is  $(-2, -1)$ , and  $a = 3$ ,  $b = 5$ , so we can use  $x = -2 - 3 \cos t$ ,  $y = -1 + 5 \sin t$ , for  $0 \leq t \leq 2\pi$ .

9. This ellipse has a graph which is the same curve in the  $xy$  plane, but it is traced out at twice the speed. At  $t = \pi$ , this parameterization has already returned to its starting point  $(9, 4)$ . This parameterization traces out the ellipse twice during  $0 \leq t \leq 2\pi$ .

10. The fact that the parameter is called  $s$ , not  $t$ , makes no difference. The minus sign means that the ellipse is traced out in the opposite direction. The graph of the ellipse in the  $xy$ -plane is the same as the ellipse in the example, and it is traced out once as  $s$  increases from 0 to  $2\pi$ .

### Problems

11. Completing the square on  $x^2 - 2x$  and  $y^2 + 4y$ :

$$\begin{aligned} \frac{1}{4}(x^2 - 2x) + y^2 + 4y + \frac{13}{4} &= 0 \\ \frac{1}{4}((x-1)^2 - 1) + (y+2)^2 - 4 + \frac{13}{4} &= 0 \\ \frac{1}{4}(x-1)^2 - \frac{1}{4} + (y+2)^2 - 4 + \frac{13}{4} &= 0 \\ \frac{(x-1)^2}{4} + (y+2)^2 &= 1. \end{aligned}$$

The center is  $(1, -2)$ , and  $a = 2$ ,  $b = 1$ .

12. Completing the square on  $x^2 + 4x$  and  $y^2 + 10y$ :

$$\begin{aligned} \frac{1}{9}(x^2 + 4x) + \frac{1}{25}(y^2 + 10y) &= -\frac{4}{9} \\ \frac{1}{9}((x+2)^2 - 4) + \frac{1}{25}((y+5)^2 - 25) &= -\frac{4}{9} \\ \frac{1}{9}(x+2)^2 - \frac{4}{9} + \frac{1}{25}(y+5)^2 - 1 + \frac{13}{4} &= -\frac{4}{9} \\ \frac{(x+2)^2}{9} + \frac{(y+5)^2}{25} &= 1. \end{aligned}$$

The center is  $(-2, -5)$ , and  $a = 3$ ,  $b = 5$ .

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12.3

Dividing by 36 to get 1 on the right:

$$\frac{9\left(x + \frac{1}{2}\right)^2}{36} + \frac{4\left(y - \frac{1}{2}\right)^2}{36} = \frac{36}{36}$$

$$\frac{\left(x + \frac{1}{2}\right)^2}{4} + \frac{\left(y - \frac{1}{2}\right)^2}{9} = 1.$$

The center is  $\left(-\frac{1}{2}, \frac{1}{2}\right)$ , and  $a = 2$ ,  $b = 3$ .

17. (a) The center is  $(-1, 3)$ , major axis  $a = \sqrt{6}$ , minor axis  $b = 2$ . Figure 12.28 shows the graph.

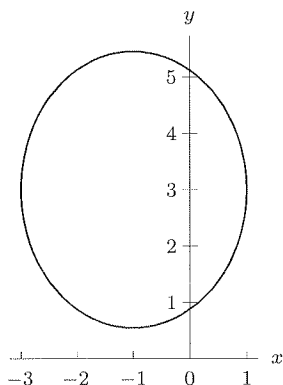


Figure 12.28

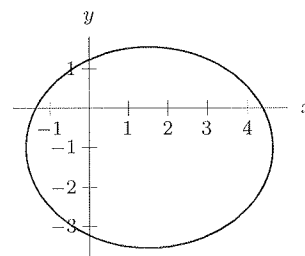


Figure 12.29

- (b) Rewriting the equation and completing the square, we have

$$2x^2 + 3y^2 - 6x + 6y = 12$$

$$2(x^2 - 3x) + 3(y^2 + 2y) = 12$$

$$2\left(x^2 - 3x + \frac{9}{4}\right) + 3(y^2 + 2y + 1) - \frac{9}{2} - 3 = 12$$

$$2\left(x - \frac{3}{2}\right)^2 + 3(y + 1)^2 = \frac{39}{2}$$

$$\frac{\left(x - \frac{3}{2}\right)^2}{(39/4)} + \frac{(y + 1)^2}{(13/2)} = 1.$$

So the center is  $(3/2, -1)$ , major axis is  $a = \sqrt{39}/2$ , minor axis is  $b = \sqrt{13}/2$ . Figure 12.29 shows the graph.

18. The center of the ellipse is at  $(h, k)$ ; both coordinates are positive here. Figure 12.30 shows  $a, b, h, k$  on the ellipse. We see that

$$0 < k < b < h < a.$$

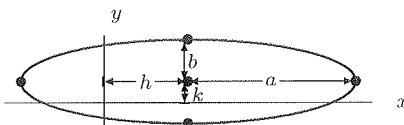


Figure 12.30

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12.3